Degradation modelling for predictive maintenance under various operating and environmental conditions

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This article deals with the estimation of the reliability and the remaining potential of equipment subject to wear whose degradation level are directly or indirectly observable. Based on accelerated non-stationary Lévy processes, reliability is estimated under various operating and environmental conditions, considering that wear level is included between the current state observed and an acceptable threshold. Among the existing models, the Variance Gamma process shows great flexibility to depict the diversity of degradation phenomena and is therefore well suited for predictive models' developments. However, its adjustment is difficult because its likelihood function includes a Bessel function in its expression and can thus have several local optima. Hybrid optimisation (global/local) then appears more precise than the local methods generally used. The industrial application case presented in the communication was carried out as part of the RYTHMS project funded by the European Union's Clean Sky2 research program. It seeks to characterise optoelectronic components through accelerated testing.

Keywords: predictive maintenance, prognosis, Remaining Useful Life, Gamma Variance process, accelerated testing.

1. Introduction

Predictive maintenance (or conditional maintenance) is getting a lot of interest, but its effective implementation struggles to materialise. Apart from diagnosis, which estimates the operating state of equipment, it involves the use of a predictive model which describes its behaviour under various operating and environmental conditions, to assess a prognosis. The difficulty lies in the model choice.

- Explanatory degradation models are little used because they require precise knowledge of the physics phenomena.
- If only the failure is observable, reliability laws, such as Weibull or lognormal, are used. They can be accelerated to take into account stress conditions, but require the observation of numerous operating times to estimate their parameters.
- If a level of wear is directly or indirectly observable, the Lévy[a] processes offer degradation models that can be accelerated and made non-stationary to vary the rate of degradation over time.
- Other models exist such as the discrete state processes of a Markovian family which grows steadily. However, they do not seem to lead to the resolution of many concrete applications.

Research [1] has recently enabled to characterise the reliability of electronic components subjected to wear, within the framework of the RYTHMS project funded by the Clean Sky2 research program of the European Union. The assessment is based on the results of accelerated degradation tests that are used to fit Gamma or Wiener-type Lévy processes. The reliability is then assessed under various operating and environmental conditions, considering that the level of degradation remains below an acceptable threshold. The degradation model is used to assess mission success chances, within reliability estimate forecasts, or to deduce RUL (Remaining Useful Life) from current state observed degradation level, within predictive maintenance.

However, the gamma process is monotonic and the Wiener one consists of the sum of a drift and a Gaussian noise, as illustrated in figure 1. A more flexible model is therefore necessary to represent certain non-monotonic trajectories, with random jumps of degradation or partial remission, like the one represented in figure 1: the Variance Gamma process [2] [3].

The work presented in the present article is part of an R&D activity, within the predictive maintenance, that aims at improving predictive reliability estimation or RUL (Remaining Useful Life) of products whose degradation levels are, directly or indirectly, observable during tests, normal use or maintenance actions, under various operating and environmental conditions. Potential applications range from simple electronic components to complex systems such as aircraft [14] or space satellites [15].

2. Lévy process

Degradation phenomena, such as wear or crack propagation, can be modelled by Lévy processes. These stochastic processes are characterised by independent increments which depend only on the time interval length (stationary). The best-known models are those of Wiener, Gamma and Compound Poisson. Initially introduced in the late 90s in the financial sector, Gamma variance process may also become relevant in the field of reliability in order to better represent certain degradation phenomena.

[a] Paul Lévy (1886 – 1971): French mathematician.

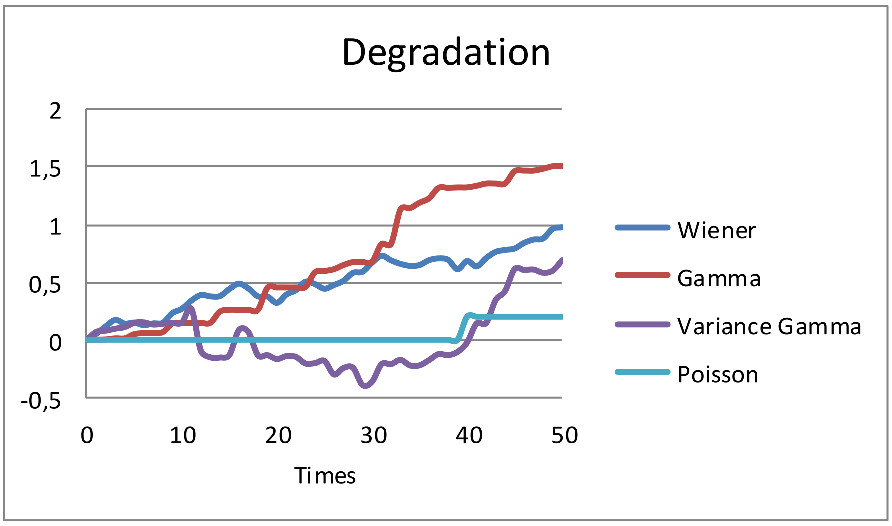


Fig. 1. Lévy process

2.1. *Accelerated non-stationary Lévy process*

In order to consider a non-constant degradation rate, the Levy process can be made non-stationary, using a timescale transformation with the power function shown in Figure 2, for example.

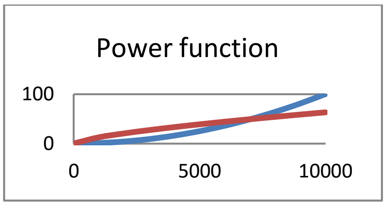


Fig. 2. Function pt^q with $0 < q < 1$ or $q > 1$

It can be accelerated to take into account stresses generated by operating and environmental conditions, as shown in Figure 3.

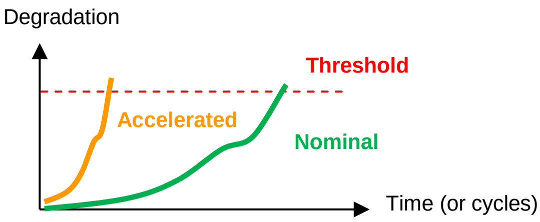


Fig. 3. Accelerated non-stationary process

The change in degradation between two instants is then calculated by replacing the time difference h by h ':

$$h' = p(AF\,(t+h))^q - p(AF\,t)^q \qquad (1)$$

With AF an acceleration factor of type Accelerated Standard of Life [4]. Representing most of the acceleration models used in the reliability field (Arrhenius, Basquin, etc.), this kind of models assume that only the scale factor of the reliability or degradation law is modified but not its shape. Moreover, if the stress level varies, Sedyakin principle [5] makes it possible to determine an equivalent acceleration factor, by integration of the acceleration factor to the stress conditions in the current state (2), that embodies the stress profile applied between two measurements.

$$AF(S_{Equivalent}) = \frac{1}{t}\int_0^t AF(S(u))du \qquad (2)$$

The degradation between two moments evolves, as follows:

$$h' = p(AF_e(t+h)\,(t+h))^q - p(AF_e(t)\,t)^q \qquad (3)$$

where $AF_e(t)$ and $AF_e(t+h)$ are respectively the equivalent acceleration factors between t_0 and t, and t_0 and t+h.

2.2. *Wiener process*

A Wiener process is a Brownian motion with drift which is expressed as follows:

$$W(t, \theta, \sigma) = b(t, \theta, \sigma) = \theta\, t + \sigma\, w(t) \qquad (4)$$

where θ is the drift, σ is the volatility and w(t) is the normal random variable with zero mean and variance t. Its evolution between two times t and t + h is modelled by a normal law with mean θh and standard deviation $\sigma\sqrt{h}$. Its probability density function with respect to the variable h has the following expression:

$$f_{b(h)}(x) = \frac{1}{\sigma\sqrt{2\pi h}} e^{-\frac{(x-\theta h)^2}{2\sigma^2 h}} \qquad (5)$$

2.3. *Gamma process*

A gamma process $\gamma(t, \mu, \nu)$ is defined by positive increments distributed according to a gamma law of expectation μh and variance νh. Its evolution between two times t and t + h is modelled by a gamma law of parameters αh and β or μ^2/ν h and μ/ν (gamma law of parameters α and β having for expectation $\mu = \alpha\beta$ and for variance $\nu = \alpha\beta^2$). Its probability density has the expression (g> 0):

$$f_{\gamma(h)}(g) = \frac{g^{\alpha h-1}\beta^{\alpha h}e^{-\beta g}}{\Gamma(\alpha h)} \qquad (6)$$

or $$f_{\gamma(h)}(g) = \left(\frac{\mu}{\nu}\right)^{\frac{\mu^2 h}{\nu}} \frac{g^{\left(\frac{\mu^2 h}{\nu}-1\right)} e^{\left(-\frac{\mu}{\nu}g\right)}}{\Gamma\left(\frac{\mu^2 h}{\nu}\right)} \qquad (7)$$

where $\Gamma(x)$ is the gamma function.

2.4. *Compound Poisson process*

A compound (or marked) Poisson process is characterised by positive increases occurring at random times which follow a Poisson distribution. The degradation path results, for example, from an accumulation of shocks without modification of the degradation level between two shocks. The increase in degradation following a shock may be constant or may depend on its order in the shocks list. This process can be combined with other Lévy processes to model certain degradation phenomena.

2.5. Variance Gamma process

A Variance Gamma process corresponds to Brownian motion with drift subjected to random time changes according to a gamma process of type $\gamma(t, \mu = 1, \nu)$:

$$X(t, \nu, \theta, \sigma) = b(\gamma(t, 1, \nu), \theta, \sigma) \qquad (8)$$

Its probability density function at time t can be expressed by means of the normal density function conditioned to the occurrence of a time g according to a gamma distribution of expectation t and variance νt:

$$f_{X_t}(X) = \int_0^\infty f_{b(g)}(x) \times f_{\gamma(t)}(g)\, dg \qquad (9)$$ or

$$f_{X_t}(X) = \int_0^\infty \frac{1}{\sigma\sqrt{2\pi g}} e^{-\frac{(x-\theta g)^2}{2\sigma^2 g}} \left(\frac{1}{\nu}\right)^{\frac{t}{\nu}} \frac{g^{\left(\frac{t}{\nu}-1\right)} e^{\left(-\frac{g}{\nu}\right)}}{\Gamma\left(\frac{t}{\nu}\right)} dg \qquad (10)$$

This density is computed in Figure 4 for a given set of parameters.

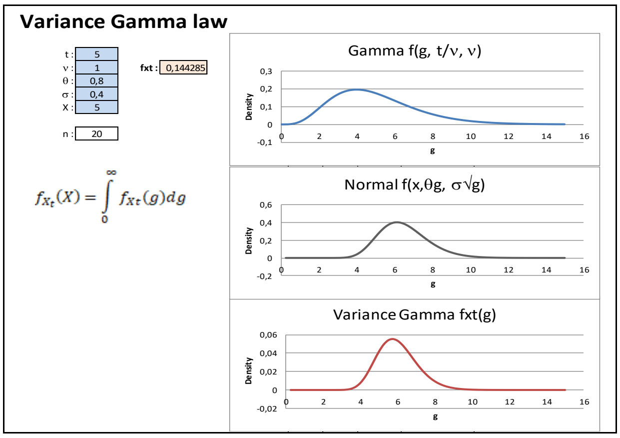


Fig. 4. Density function of the Variance Gamma process

This probability density can also be expressed from a second type Bessel function [8]. In the general case of a Variance Gamma process following $X(t, \nu, \theta, \sigma) = b(\gamma(t, \mu, \nu), \theta, \sigma)$, it is equal to :

$$f_{X_t}(X) = \int_0^\infty \frac{1}{\sigma\sqrt{2\pi g}} e^{-\frac{(x-\theta g)^2}{2\sigma^2 g}} \left(\frac{\mu}{\nu}\right)^{\frac{\mu^2 t}{\nu}} \frac{g^{\left(\frac{\mu^2 t}{\nu}-1\right)} e^{\left(-\frac{\mu g}{\nu}\right)}}{\Gamma\left(\frac{\mu^2 t}{\nu}\right)} dg \qquad (11)$$

or

$$f_{X_t}(X) = \frac{2e^{\theta\frac{(x-\mu)}{\sigma^2}}}{\nu^{\frac{t}{\nu}}\sigma\sqrt{2\pi}\Gamma\left(\frac{t}{\nu}\right)} \left(\frac{(x-\mu)^2}{\frac{2\sigma^2}{\nu}+\theta^2}\right)^{\frac{t}{2\nu}-\frac{1}{4}} K_{\frac{t}{\nu}-\frac{1}{2}}\left(\frac{1}{\sigma^2}\sqrt{(x-\mu)^2\left(\frac{2\sigma^2}{\nu}+\theta^2\right)}\right) \qquad (12)$$

Where $K\alpha(x)$ is the second type Bessel function and $\Gamma(x)$ is the gamma function [8]. The latter expression is not defined for x = 0. It is equivalent to the following expression that is most often used for the Gamma variance distribution:

$$f(x) = \frac{\gamma^{2\lambda}|x-\mu|^{\lambda-\frac{1}{2}} K_{\lambda-\frac{1}{2}}(\alpha|x-\mu|)}{\sqrt{\pi}\,\Gamma(\alpha)\,(2\alpha)^{\lambda-1/2}} e^{\beta(x-\mu)} \qquad (13)$$

where μ is the location parameter

β is the asymmetry parameter equal to $\theta/\sigma2$

λ the shape parameter equal to t/ν

α is equal to $1/\sigma^2\sqrt{2\sigma^2/\nu + \theta^2}$

γ is equal to $\sqrt{\alpha^2 - \beta^2}$

Note that the Gamma variance process is also defined as a difference between two independent gamma processes with the same parameter α:

$$X(t, \nu, \theta, \sigma) = \gamma(t, \mu_p, \nu_p) - \gamma(t, \mu_n, \nu_n) \qquad (14)$$

with $\alpha_p = \mu_p^2/\nu_p = \alpha_n = \mu_n^2/\nu_n = 1/\nu$

The value of the parameters can then be calculated by equalizing the respective characteristic functions which completely define their probability distribution [3]:

$$\mu_p = \frac{1}{2}\sqrt{\theta^2 + \frac{2\sigma^2}{\nu}} + \frac{\theta}{2} \qquad \mu_n = \frac{1}{2}\sqrt{\theta^2 + \frac{2\sigma^2}{\nu}} - \frac{\theta}{2}$$

$$\nu_p = \left(\frac{1}{2}\sqrt{\theta^2 + \frac{2\sigma^2}{\nu}} + \frac{\theta}{2}\right)^2 \nu \qquad \nu_n = \left(\frac{1}{2}\sqrt{\theta^2 + \frac{2\sigma^2}{\nu}} - \frac{\theta}{2}\right)^2 \nu$$

The Variance Gamma process gives an analytical expression of its moments [3]:

Expectation value: $E[X(t)] = \theta\, t$

Variance : $E[(X(t) - E[X(t)])^2] = (\theta^2\, \nu + \sigma^2)\, t$

Asymmetry coefficient $\times$ Variance$^{3/2}$:

$$E[(X(t) - E[X(t)])^3] = (2\,\theta^3\,\nu^2 + 3\,\sigma^2\,\theta\,\nu)\, t$$

Flattening coefficient (Kurtosis) $\times$ Variance^2 :

$E[(X(t) - E[X(t)])^4] = (3\,\sigma^4\,\nu + 12\,\sigma^2\,\theta^2\,\nu^2 + 6\,\theta^4\,\nu^3)\, t + (3\,\sigma^4 + 6\,\sigma^2\,\theta^2\,\nu + 3\,\theta^4\,\nu^2)\, t^2$

4.1. Calculation and simulation

Monte-Carlo simulation of a Variance Gamma process consists of simulating the variable g by a gamma law and then the variable x by a normal law, which gives in Excel:

g = GAMMAINV(RAND(),t/nu,nu)

x = Téta*g+Sigma*SQRT(g)*NORM.S.INV(RAND())

or in other words:

x = NORM.INV(RAND();Téta*g;Sigma*SQRT(g))

Two macro-functions were developed to calculate the probability density of the Variance Gamma process, one by integration according to formula 11 and the other with the Bessel function according to formula 12.

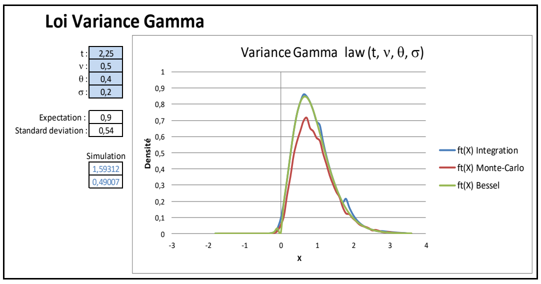


Fig. 5. Simulated and computed density

Figure 5 shows a good correspondence between the probability density curves obtained by integration and by means of the Bessel function, for a given set of parameters. It is also quite close to a curve obtained by Monte-Carlo simulation (16,000 draws). The probability density can also be computed, in Excel 2019, using BESSELK(X, N) function where N (α) is positive and truncated when it is not an integer.

Figure 6 shows the truncation effect on the calculated probability density.

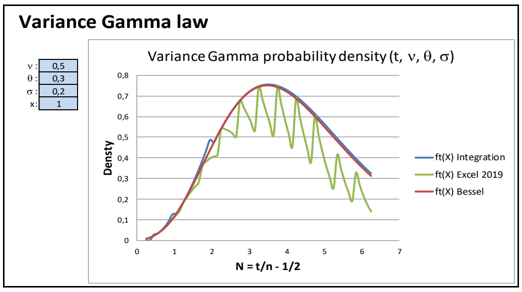


Fig. 6. Effect of the integer truncation of α

Figure 7 shows the different influences the parameters of the Variance Gamma law have on the distribution.

Figure 8 presents the adjustment of the Variance Gamma with the help of a macro-function involving a Bessel function, which is much faster than the method using integration. A stationary process was first adjusted from a degradation trajectory simulated over 200-time steps. The parameters used for the simulation are roughly retrieved by the adjustment. A non-stationary process was then adjusted from a trajectory of 400 elementary degradation steps. The parameters used for the simulation are once again retrieved by the adjustment (q is the power function parameter).

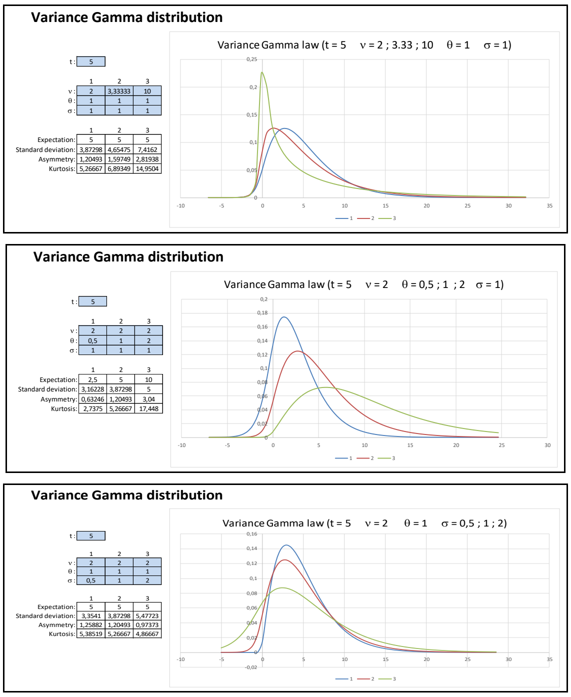


Fig. 7. Parameters influence on the distribution

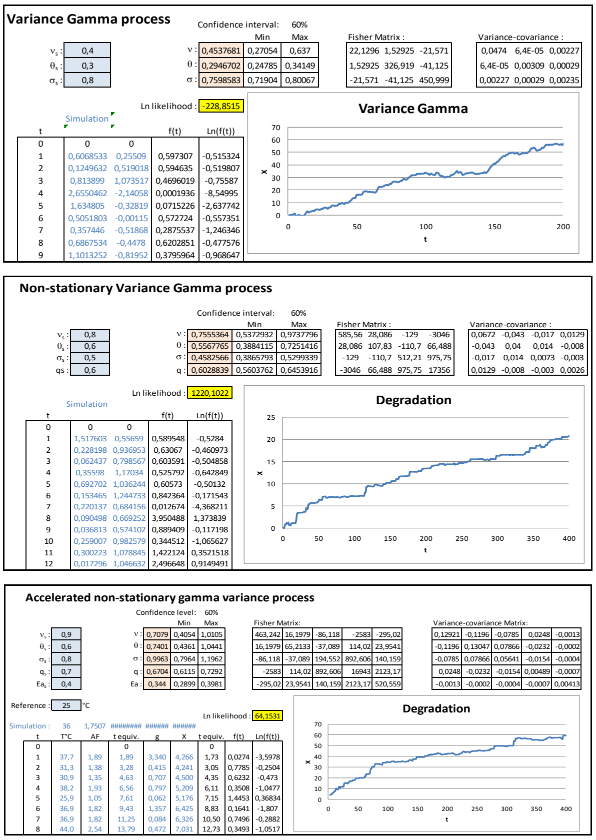


Fig. 8. Variance Gamma adjustment

An accelerated non-stationary process was finally adjusted from a trajectory of 400 elementary degradation steps at different temperatures (Ea is the Arrhenius model activation energy). The parameters used for the simulation are once again roughly the same and almost all within the 60% confidence interval.

The multi-parameter adjustment covers both the degradation model and the accelerating factors. It requires a high-performance optimisation method to be able to overcome any local optima. The adjustment was carried out here using Gencab[b], a tool based on a global/local hybrid optimisation method and which provides asymptotic confidence intervals from the Fisher matrix calculated thanks to a numerical method [6] [7].

In addition to the randomness of the simulation, it should be noted how important the accuracy of the computation of the probability density which must be similar for every observation (x, t). It cannot be obtained correctly using a Bessel function with truncated parameters. Moreover, the increase of the number of model parameters entails a larger amount of observed data entries in order to be able to obtain good estimate of model parameters.

2.7. Reliability model

The definition of a degradation acceptability threshold enables the move from a degradation model to a reliability model as illustrated in figure 3. The reliability corresponds to the law of first crossing time of a threshold z_s:

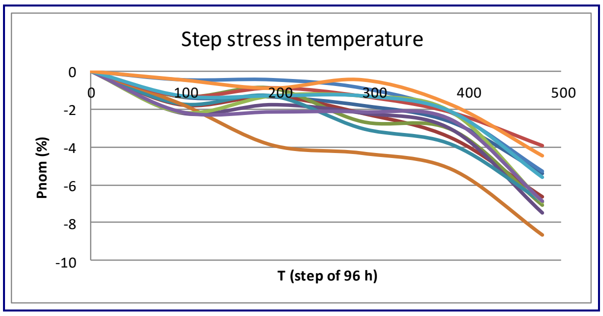
$$R(t) = P(Z(t) \leq z_S) \text{ or } R(t-t_0) = P(Z(t-t_0) \leq z_S - z_0) \quad (15)$$

In the case of a Gamma process, this law is the Gamma law since it is monotonic. The MRL (Mean Residual Life), which is the expectation of the RUL (Remaining Useful Life), can be estimated by integrating the reliability function considering the initial degradation level and the threshold of the acceptable operation domain. There is no law describing the first threshold crossing time for non-monotonic random processes such as Wiener or Variance Gamma, but the reliability, as well as the MRL, can be estimated by Monte Carlo simulation.

3. Application

As part of the RYTHMS project, an optoelectronic module with embedded VCSELs (Vertical Cavity Surface Emitting Laser), has been characterised for optical interconnections in aeronautical and space applications. Its reliability was estimated from results of tests accelerated in temperature and current following step stresses, presented in Figure 9. The observable degradation is that of the optical power, the admissible level of which is greater than 90% of its initial value. 171 statistical data were recorded, and hence, there are between 8 and 9 degradation values per component part of a batch of about 20 pieces of the same type.

As transient improvements are observed on the curves of Figure 9, the degradation cannot be modelled by a monotonic process such as the Gamma process. Therefore, a Wiener model non-stationary and accelerated was chosen for the estimation and was fitted by the maximum likelihood method as shown in Figure 10 (only the first two components are shown here among the 20).



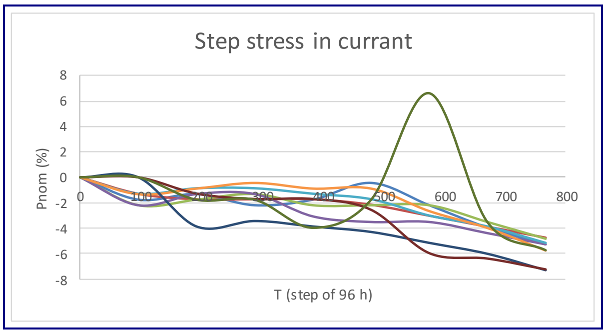


Fig. 9. Degradation trajectories

The adjustment covers both the degradation model and the accelerating factors. It requires a high-performance optimisation method to be able to process all the parameters and exceed any local optimum. The 5 parameters estimated from data are θ and σ for the normal law, q of the power function pt^q (with p = 1), Ea the activation energy of Arrhenius' law and n the parameter of an acceleration factor in the current of reverse power type. The global acceleration factor expression is, as follows:

$$AF = \left(\frac{i}{i_{ref}}\right)^n e^{-\frac{Ea}{K}\left(\frac{1}{T_{ref}} - \frac{1}{T}\right)} \quad (16)$$

Where :

- i and i_{ref} (5mA) are currents respectively in test and reference conditions.

- T and T_{ref} (40°C) are temperatures respectively in test and reference conditions in °K (T°C + 273),

- K is the Boltzmann constant.

The fitted model was then subjected to a Monte Carlo simulation to obtain the reliability curve as well as the expectation and quantiles of the Remaining Useful Life. A reliability law equivalent to the threshold zs – z0 (Weibull or log-normal usually used to quantify the reliability of laser diodes) can then be fitted by the method of least squares to simplify higher level reliability evaluations.

For comparison purposes, an accelerated non-stationary Variance Gamma process was fitted from the same degradation trajectories (figure 11), as well as an deterministic accelerated power model of type $p(AF\ t)^q$ (Figure 12). The reliability estimates appear to be much less conservative, mainly due to the lower values of the acceleration parameters Ea and n.

b Generic tool developed and marketed by Cab Innovation, based on the Nelder Mead algorithm coupled with genetic algorithms.

Non stationary Wiener process accelerated in temperature and current

σ :	0,047004351
θ :	0,000836948
q :	1,016058545
Ea :	0,197364641
n :	1,928459679

Tj_{REF}	i_{REF}
40	5

Ln Likelihood: -925,583839

Confidence rate: 60%

Confidence interval:

		Min	Max
σ :	0,047	0,03628	0,05773
θ :	0,02473	0,02434	0,02513
q :	1,01606	0,94774	1,08438
Ea :	0,19736	0,1624	0,23233
n :	1,92846	1,60938	2,24753

Fisher's Matrix:

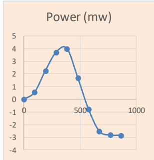
154794	0,31836	32342,9	20831,8	2421,7
0,31836	1,1E+08	825431	558676	67272,9
32342,9	825431	13421,4	8929,17	1048,71
20831,8	558676	8929,17	6624,11	672,762
2421,7	67272,9	1048,71	672,762	90,0511

Variance-covariance matrix:

0,00016	5,7E-06	-0,0008	0,0001	0,00047
5,7E-06	2,2E-07	-3E-05	3E-06	8,5E-06
-0,0008	-3E-05	0,00659	-0,002	-0,0166
0,0001	3E-06	-0,002	0,00173	0,00559
0,00047	8,5E-06	-0,0166	0,00559	0,14373

E165

t	Tj	i (ma)	FA	t equivalent	Pnom [%]	f(t)	Ln(f(t))	Simulation Power (mw)
0	50,3452	8		0	0			0
96	50,351	8	3,12874403	300	-1,754386	0,10400447	-2,26332136	0,555093991
192	85,35035	8	6,24916964	900	-1,3157895	0,23306424	-1,45644116	2,258517183
288	100,34945	8	8,07865176	1676	-2,1929825	0,28411127	-1,25838931	3,723227882
384	110,3478	8	9,48043148	2586	-1,754386	0,18064859	-1,71120162	4,008308352
480	120,34825	8	11,0356564	3645	-0,4385965	0,08811027	-2,4291662	1,714844137
576	130,34665	8	12,749244	4869	-2,1929825	0,21394912	-1,54201705	-0,78487403
672	140,34745	8	14,6268914	6273	-3,9473684	0,20546459	-1,58248155	-2,5238526
768	150,3484	8	16,672526	7874	-5,2631579	0,19449393	-1,63735433	-2,82543676
864	50,3499	8	3,12866864	8174	-6,1403509	0,3636555	-1,0115483	-2,83244552



E166

t	Tj	i (ma)	FA	t equivalent	Pnom [%]	f(t)	Ln(f(t))	Simulation Power (mw)
0	50,3444	8		0	0			0
96	50,35035	8	3,12869948	300	-1,2931034	0,22955109	-1,47162967	#NOMBRE!

Non stationary Wiener process accelerated in temperature and current

σ :	0,04700435
θ :	0,00083695
q :	1,01605854
Ea :	0,19736464
n :	1,92845968

Tj_{REF}	i_{REF}
40	5

Tj (°C)	i (ma)	AF
40	5	1

	Simulation	Mean	Mean inf 60%	Mean sup 60%	Q10 60%	Q90 60%
Residual Life:	6500	10685	10648	10721	5000	18000

t	t equivalent	Simulation	Mean	M-60%	M+60%	Q10 60%	Q90 60%	Expectation
0	0	0	0	0	0	0	0	0
500	500	-0,5308822	-0,4588	-0,4661	-0,4514	-1,8596	0,9560597	-0,462392
1000	1000	-2,6908438	-0,9209	-0,9315	-0,9104	-2,9541	1,1249345	-0,935135
1500	1500	-2,8244181	-1,3969	-1,4099	-1,384	-3,8822	1,1251432	-1,411865
2000	2000	-4,1028957	-1,8736	-1,8885	-1,8586	-4,7573	0,9950306	-1,891203
2500	2500	-3,8445296	-2,3547	-2,3715	-2,3379	-5,5963	0,9020144	-2,37249
3000	3000	-4,2174292	-2,8473	-2,8659	-2,8287	-6,4711	0,7326204	-2,855336
3500	3500	-4,5954608	-3,318	-3,3381	-3,298	-7,2243	0,5350293	-3,339482
4000	4000	-5,1125638	-3,8073	-3,8288	-3,7858	-7,9261	0,3946707	-3,824743
4500	4500	-6,5028752	-4,2798	-4,3026	-4,2571	-8,6743	0,1585462	-4,310982
5000	5000	-6,5816903	-4,7595	-4,7836	-4,7354	-9,4451	-0,100092	-4,798092
5500	5500	-9,3639819	-5,2403	-5,2656	-5,215	-10,168	-0,346175	-5,285985

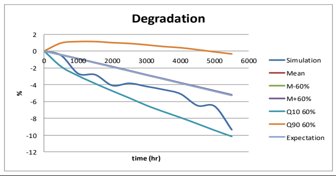


Fig. 10. Adjustment and reliability estimation by Wiener process

Non stationary Variance gamma process accelerated in temperature and current

ν :	390,3420998
θ :	-0,00171874
σ :	0,053427276
q :	1,098461831
Ea :	0,031942469
n :	1,425345303

Tj_{REF}	i_{REF}
40	5

Ln Vraisemblance : -17700

Confidence rate: 60%

Confidence interval:

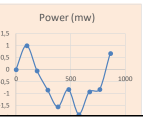
		Min	Max
ν :	390,3421	163,70164	616,98256
θ :	-0,001719	-0,002642	-0,000795
σ :	0,0534273	0,0389771	0,0678775
q :	1,0984618	1,0173091	1,1796146
Ea :	0,0319425	-0,007011	0,0708963
n :	1,4253453	1,0938641	1,7568265

Fisher's Matrix:

8,228E-05	-0,608589	-0,16102	-0,277551	-0,181728	-0,023254
-0,608589	29963673	281761,76	-343192,6	-269687	-35534,83
-0,16102	281761,76	64269,48	10061,214	8053,5296	933,53024
-0,277551	-343192,6	10061,214	7722,6801	5965,0744	759,18559
-0,181728	-269687	8053,5296	5965,0744	5127,2112	572,72059
-0,023254	-35534,83	933,53024	759,18559	572,72059	81,869148

E165

t	Tj	i (ma)	FA	t equivalent	Pnom [%]	f(t)	Ln(f(t))		Simulation Power (mw)
0	50,3452	8		0	0				0
96	50,351	8	2,0295811	195	-1,754386	#REF!	-100	1520,2	1,002034765
192	85,35035	8	2,27003782	413	-1,3157895	#REF!	-100	266,03	-0,05098882
288	100,34945	8	2,36636366	640	-2,1929825	#REF!	-100	68,461	-0,8430641
384	110,3478	8	2,42844329	873	-1,754386	#REF!	-100	155,44	-1,56626308
480	120,34825	8	2,48888532	1112	-0,4385965	#REF!	-100	299,71	-0,83658483
576	130,34665	8	2,54771206	1357	-2,1929825	#REF!	-100	1566,5	-1,87075651
672	140,34745	8	2,60499709	1607	-3,9473684	#REF!	-100	648,8	-0,92949598



Non stationary Variance Gamma accelerated in temperature and current

ν :	390,3421
θ :	-0,0017187
σ :	0,05342728
q :	1,09846183
Ea :	0,03194247
n :	1,4253453

Tj_{REF}	i_{REF}
40	5

Tj (°C)	i (ma)	AF
40	5	1

	Simulation	Moyenne	Mean inf 60%	Mean sup 60%	Q10 60%	Q90 60%
Residual Life:	2500	2668	2658	2679	1500	4500

t	t equivalent	Simulation		Mean	M-60%	M+60%	Q10 60%	Q90 60%	Expectation
0	0	0	0	0	0	0	0	0	
500	500	1142,72985	-1,385285	-1,5402	-1,556286	-1,524181	-4,0474	0,5472497	-1,584621359
1000	1000	544,632058	-2,7698136	-3,3376	-3,361231	-3,3139	-7,0667	-0,057617	-3,393090677
1500	1500	1081,98203	-4,8893675	-5,1877	-5,217325	-5,158093	-9,7781	-0,95484	-5,296939258
2000	2000	1198,77201	-8,1195399	-7,1383	-7,173006	-7,103495	-12,58	-2,079118	-7,265498645
2500	2500	736,588317	-8,7152654	-9,1555	-9,195172	-9,11582	-15,168	-3,310676	-9,283620449
3000	3000	1481,75614	-11,637549	-11,199	-11,24303	-11,15585	-17,896	-4,678536	-11,34213868
3500	3500	1678,65365	-16,048489	-13,314	-13,3611	-13,26669	-20,544	-6,274509	-13,43486931
4000	4000	906,396341	-16,790505	-15,45	-15,50058	-15,39946	-23,297	-7,858257	-15,55734155
4500	4500	954,465327	-19,855493	-17,591	-17,64479	-17,53648	-25,892	-9,486362	-17,70616389
5000	5000	1396,36117	-19,738707	-19,772	-19,82899	-19,71453	-28,545	-11,19318	-19,87867057
5500	5500	418,818102	-19,942925	-21,926	-21,98641	-21,86647	-31,32	-13,0276	-22,07270817

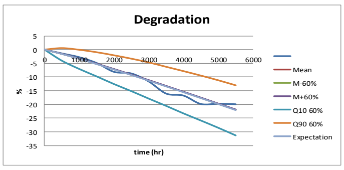


Fig. 11. Adjustment and reliability estimation by Variance Gamma process

Accelerated power function

p :	-4,4604E-06
q :	1,314102259
Ea :	0,295811812
n :	3,469258433

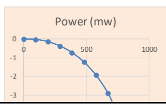
Tj_{REF}	i_{REF}
40	5

$\Sigma\varepsilon^2$: 490,8351209

BIAS	MAE	RMSE
-0,11767	1,072	1,69422

E165

t	Tj	i (ma)	FA	t equivalent	Pnom [%]	Δx	ε^2	Simulation Power (mw)
0	50,3452	8		0	0			0
96	50,351	8	7,25476048	696	-1,754386	-0,0242789	2,99327032	-0,02427894
192	85,35035	8	20,4617854	2661	-1,3157895	-0,1170365	0,308728051	-0,14131547
288	100,34945	8	30,0666831	5547	-2,1929825	-0,2297672	0,419160096	-0,3710827
384	110,3478	8	38,2152365	9216	-1,754386	-0,3519883	0,625024349	-0,72307103
480	120,34825	8	47,9858342	13822	-0,4385965	-0,5086994	3,328759683	-1,23177044
576	130,34665	8	59,5755088	19542	-2,1929825	-0,709743	1,091278984	-1,94151341



Accelerated power function

t_0 :	5
X_0 :	0

p :	-4,46E-06
q :	1,31410226
Ea :	0,29581181
n :	3,46925843

Tj_{REF}	i_{REF}
40	5

Mean Residual Life : 68025

Tj (°C)	i (ma)	AF
40	5	1

Fig. 12. Adjustment and reliability estimation by power function

Life test results carried out over longer periods (2000 h) were obtained after these first estimates. These data (n = 70) were used to assess the prediction quality of the different models, by means of 3 indicators as shown in figure 13:

- the bias: $Bias = \frac{\sum_{i=1}^{n}(\widehat{y_i}-y_i)}{n}$

- the mean absolute error: $MAE = \frac{\sum_{i=1}^{n}|(\widehat{y_i}-y_i)|}{n}$

- the root mean square error: $RMSE = \sqrt{\frac{\sum_{i=1}^{n}(\widehat{y_i}-y_i)^2}{n}}$

Comparison of predictive models

	Wiener			Variance Gamma			Power		
	BIAS	MAE	RMSE	BIAS	MAE	RMSE	BIAS	MAE	RMSE
	1,827	2,982	4,658	2,511	3,585	5,292	1,18	2,775	4,596

i_{REF}	Tj_{REF}
5	40

AC197

t	i (ma)	Tj	Pnom [%]	$t_{equivalent}$				$t_{equivalent}$				$t_{equivalent}$			
0	10	84,9415	0	0				0				0			
168	10	84,94275	-2,13675	1603	-0,63	0,627	0,393	523,6	-0,47	0,47	0,221	7374	-1,6	1,597	2,551
500	10	84,943	-2,5641	4770	2,636	2,636	6,951	1558	3,429	3,429	11,76	21947	1,295	1,295	1,676
1000	10	84,9495	-13,6752	9540	-6,43	6,435	41,41	3116	-4,81	4,808	23,11	43895	-7,75	7,749	60,05

Fig. 13. Comparison of the predicted models

The predictions appear here to be really promising, especially since the model is simple and rigid, which can be explained with the insufficiency of data used for the adjustment: the models flexibility partly overcomes the effect of the stress conditions (171 statistical data used instead of 400 in the examples in Figure 8).
The processing of this application case will be continued with the imminent availability of new test data and a better distribution between the data used for the adjustment (80% drawn randomly) and those used to develop the quality indicators for the predictions (20%).

4. Considering the effect of maintenance

Several types of model [9][10] aim at characterising the repair effect in order to optimise maintenance and any useful lifetime of equipment.
Between remanufacturing and no effect on ageing, a first class of imperfect repair models is based on a reduced failure rate:

$$\lambda(t_{r+}) = q\lambda(t_{r-}) \text{ with } 0 \leq q \leq 1) \quad (17)$$

A second class of models suggest reducing the age of the equipment. The rejuvenation effect may be proportional to the time elapsed since the previous maintenance action (GRP or Kijima type 1 model). The virtual age of the equipment just after the r^{th} maintenance action is equal to:

$$A_r = A_{r-1}+q(t_r-t_{r-1}) = A_{r-2}+q(t_{r-1}-t_{r-2})+q(t_r-t_{r-1})$$

$$A_r = qt_r \quad (18)$$

The rejuvenation effect can also be proportional to the virtual age of the equipment (model GRP type 2 orARA∞). After the maintenance action, it is equal to:

$$A_r = q(A_{r-1}+t_r-t_{r-1}) = q(q(A_{r-2}+t_{r-1}-t_{r-2})+t_r-t_{r-1})$$

$$A_r = \Sigma_1^r q^{r-i+1}(t_i - t_{i-1}) \quad (19)$$

Imperfect repair models based on age reduction can be used with degradation models, as shown in Figure 14.

A third class of model can also be imagined by considering that the maintenance actions improve the level of degradation of the equipment, in proportion to its current state.

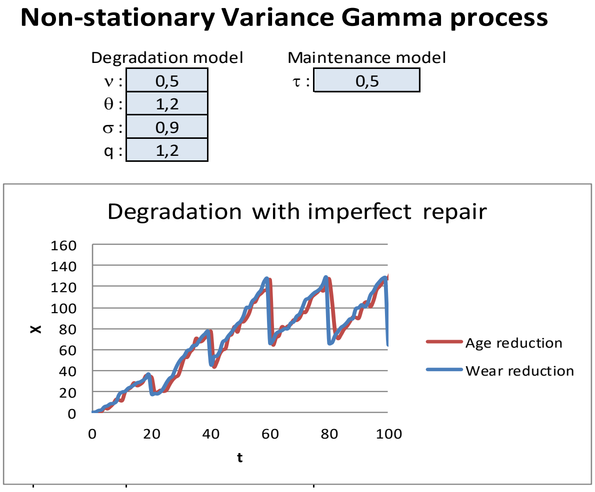


Fig. 14. Imperfect repair simulation

Moreover, the decision-making of predictive maintenance actions must be based on a degradation threshold that presents margin with the one leading to failure, especially when the level of degradation is not observable online. This margin can be optimised by calculating the expected maintenance cost at each cycle of operation and repair. This includes the predictive actions cost Cp and the corrective actions cost Cc:

$$E|C_{cycle\ i}| = \frac{C_p + (C_c - C_p)(1 - R_{Z_s}(MRL(Z_{s-margin}))}{MRL(Z_{s-margin})} \quad (20)$$

Where the MRL (Mean Residual Life), which is the expectation of the RUL (Remaining Useful Life), is evaluated at the threshold value Z_s minus the margin and $R_{Zs}(t)$ is the reliability at the threshold Z_s estimated for this value of MRL.

5. Conclusion

Lévy processes are used to model equipment degradation and to estimate their reliability or remaining potential under various utilisation and environment conditions. This prognosis is based on the observation of the degradations evolution in tests or during operation, which is statistically much richer than the observation of simple failure time. Weibull or lognormal type equivalent reliability models can then be obtained for an acceptable degradation domain.

Still little used in the reliability field, the Variance Gamma process has great flexibility in representing the diversity of degradation phenomena and is therefore well suited for building predictive models. However, its adjustment is all but simple because its likelihood function includes a Bessel function in its expression and can hence have several local optima. Hybrid optimisation (global/local) appears therefore to be more suitable than the local methods generally used. The likelihood function must also be precise and homogenous between the various observations. Comparison between different adjustments is easy because the best one is the one with the highest likelihood to the observations, when the expression of the probability density, specific to each of the models, is known. Regarding the quality of the model, this can be measured by indicators of bias (MAE) or variance (RMSE) of the estimates compared to a remainder of observations not used for the adjustment.

Perfect is the enemy of good when a sophisticated model, capable of considering all behavioural aspects, is not fitted with sufficient statistical data under various conditions of use and environment. Nonetheless, to be suitable for the prognosis, the direct or indirect observability of degradations, in testing and during operation, must be considered early on during the design of new products.

Moreover, imperfect repair models by reduction of age can be used with degradation models and a new class of models can be imagined by considering that maintenance actions improve the level of degradation of equipment, in proportion to their current state.

The work presented here will soon be included in the update of reference books [11] [12] [13]. Considering their potential, they would deserve to be completed and further explored using a collaborative framework of some sort which would be the only to answer the diverse challenges.

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