OPTIMIZATION AND RECURSIVE SIMULATION MODELLING

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ABSTRACT: This communication presents an original method of modelling for discrete states systems, possibly hybrid, as well as an original technique of coupling between optimization and Monte-Carlo simulation, which allows decreasing very significantly the computing times (time divided by 30 approximately).

1 INTRODUCTION

Used in economic or environmental forecast, the recursivity allows simplifying the behavioral models but seems not very used in the Safety and Reliability field. These models describe the behavior of a discrete states system only between two current moments, corresponding to a time incrementing (time-simulation) or to the occurrence of particular events (event-simulation), such as random state transition or thresholds crossing by continuous variables. The treatment of the simulation model just consists in re-injecting to the input, the state of the system at output, starting from an initial state, as many times as it is necessary to take into account all the system's mission duration.

In other hand, the coupling between optimization and stochastic simulation, which consists in seeking an optimal system parameters configuration starting from the evaluation results obtained by Monte-Carlo simulation, is very constraining in terms of processing duration. At first approximation, the number of simulations to be performed is equal to the number of evaluations necessary to ensure convergence, multiplied by the number of simulations required by the evaluation precision. However this duration can significantly be decreased by the choice of a strategy consisting in varying the precision of each configuration evaluation, according to the results of a coarse evaluation carried out beforehand.

The object of this communication relates to the implementation of the recursivity and such a coupling by generic simulation an optimization tools.

In order to show the capacity of this recursive technique and to demonstrate the coupling effectiveness on a real case, a problem of the space domain will be presented. This one relates to the deployment and the renewal of a constellation of Earth observation satellites.

2 RECURSIVE SIMULATION MODEL

Included in an Excel-based simulation tool (SIMCAB), this method of evaluation of discrete states systems is illustrated on the Figure 1 (Cabarbaye and all 2005).

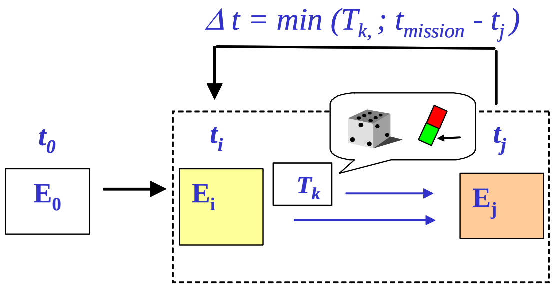


Figure 1. Recursive Model

It consists in defining a generic transition between a state Ei (at ti) and a state Ej (at tj). This transition is built by means of logical operators and of calculation between both states defined in cells of the spreadsheet. As an example, the figure 2 shows a passive redundancy M among N with a S size spare stock. The tool algorithm copies the Ej state into the Ei state during all the mission time, starting from the initial E0 state (at t0). The time slot between ti and tj is the duration of two events following each other. This duration is defined as the smallest computed value, at the current time, among the time increments Tk corresponding to system status random changes or to the overstep of thresholds by continuous parameters. Twenty probability law random functions (L_Exp, L_Wei...) are proposed by the tool to define the Tk values, with possibility of probability law adjustment starting from experimental data.

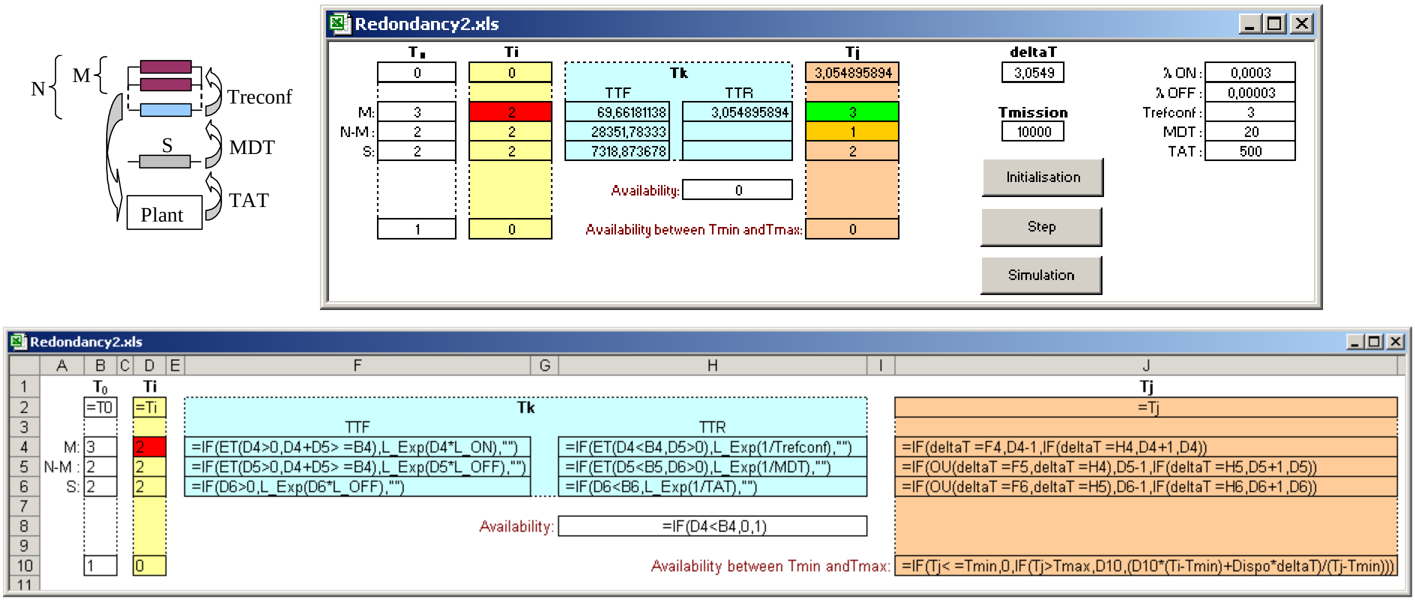


Figure 2. Passive redundancy M among N with a S size spare stock

The considered systems can be Markovian or not (with influence of the date of preceding events) and possibly of hybrid type, defined by dependences between continuous and stochastic parameters (Labeau 2003) (Castagna 2003) (Iung and all 2003).
The simulation can be done with a step by step mode in order to validate the model or for a complete mission that is re-processed numerous times depending on the targeted results precision. By means of the spreadsheet functionalities, the ergonomics of the model can be easily improved by replacing the names of cell by names of parameters, by using conditional formats (colors depending on the component state) or by coupling parameters with objects (continuous value compared to a threshold). It is thus easy to carry out an animated representation of the system, which can preserve its topology (that of a telecommunications or transportation network for example), to facilitate its validation. The example of figure 3 shows such a simulation model performed for a hybrid system test case of IMDR-ESRA (Dutuit 03) concerning a mechanism of regulation of a tank level. Other applications of this method, relating to Markovian and nonMarkovian systems, are subject of another ESREL 2006 article (FAURE and all 2006).
The model built by this recursive method is only based on logic and calculation and does not include any symbolic object. Thus, one of its originalities is to propose an alternative to the use of other behavioral methods, such as the Petri nets, which do not constitute the ideal solution for all the problems. Refusing the polemic on the comparative advantages of one solution versus the others, we will let the reader make his choice to solve new problems. According to the experiment of each one and of the specificity of the problem, the effort of modeling required and the validity of the model carried out are only the good criteria of choice.

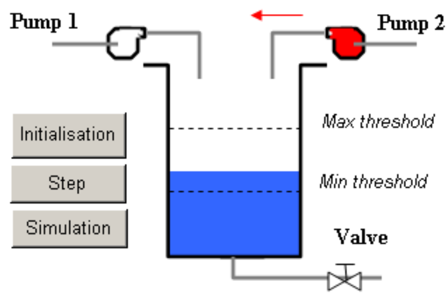


Figure 3. Animated representation

3 OPTIMIZATION COUPLING

An original coupling technique between optimization and simulation algorithms allows decreasing the processing time (Cabarbaye and all 2006). Included in an Excel-based optimisation tool (GENCAB), this technique is very efficient and divides the processing time by 30 approximately on several test examples. Based on a hybrid method associating Genetic Algorithms (Goldberg 1994), Differential Evolution (Feoktistov 2004) and nonlinear Simplex (Nelder Mead algorithm), the principle of this generic tool is illustrated by figure 4 (Cabarbaye 2003). Composed of various parameters (genes) of type real, integer or binary, the chromosomes are subjecte to random mutation, crossings and differential evolutions (summation of a gene of chromosome with the difference between same genes of two other chromosomes). After selection, the best elements of the population can be improved at the local level by several steps of Simplex. This hybridization of total and local techniques, which can be possibly parameterized, allows making the tool robust to the diversity of the problems defined by the user on a sheet of spreadsheet. Thus the Differential Evolution will be generally more effective to treat a convex function but will present the disadvantage, for others, simultaneously to exploit the whole of genes.

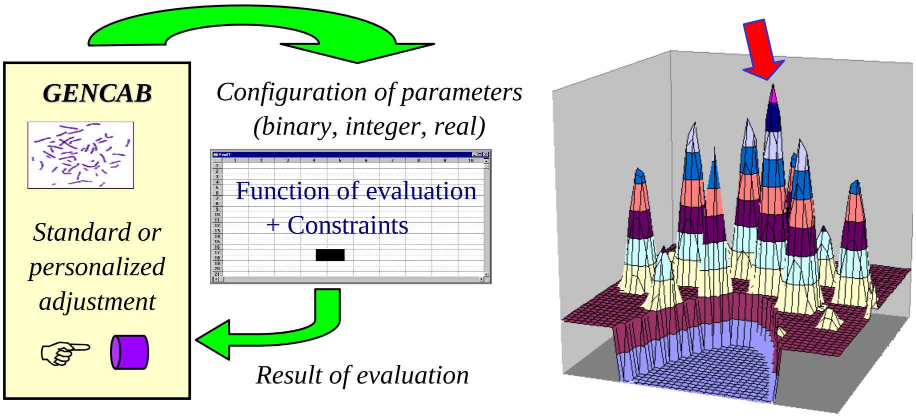


Figure 4. Principle of the tool

The principle of the coupling between optimization and simulation consists in performing a rough estimation of each solution (50 simulations of the mission for example) before estimating them again with a higher precision according to first obtained results (between 50 to 2000 simulations for example). This coupling has to ensure the same probability of inappropriate rejection for each solution. That leads to a condition between respective values Ni and Nj, of the number of simulations to evaluate two candidates i and j, according to the average and the standard deviation obtained by the rough-estimation limited to N_0 simulations.

$$Ni/Nj = [(M-m_{i0})*\sigma_{j0}/ (M-m_{j0})*\sigma_{i0}]^2$$

This condition results directly from the central limit theorem. Found in the scientific literature (Chen and all 2000), the algorithm OCBA (Optimal Computing Budget Allowance) uses this same principle to seek an optimal value among a limited number of p candidates.

$$\frac{N\hat{\imath}}{Ni} = \sigma\hat{\imath}\sqrt{\sum_{j=1,\, j\neq\hat{\imath}}^{p} \frac{1}{\sigma j^2}\rho_{ij}^2} \qquad i \neq \hat{\imath}$$

$$\rho_{ij} = \left(\frac{\sigma j/\Delta j}{\sigma i/\Delta i}\right)^2 \qquad i,\, j \in 1,2\ldots,\, p \qquad i,\, j \neq \hat{\imath}$$

$$\Delta i = J\hat{\imath} - Ji$$

Figure 5. OCBA Algorithm

At each iteration k, this one performs N news simulations distributed according to ratio indicated of figure 5, with î the best current solution found during the iteration k-1; with Ji (average) and σi (standard deviation) resulting from the evaluation of i.

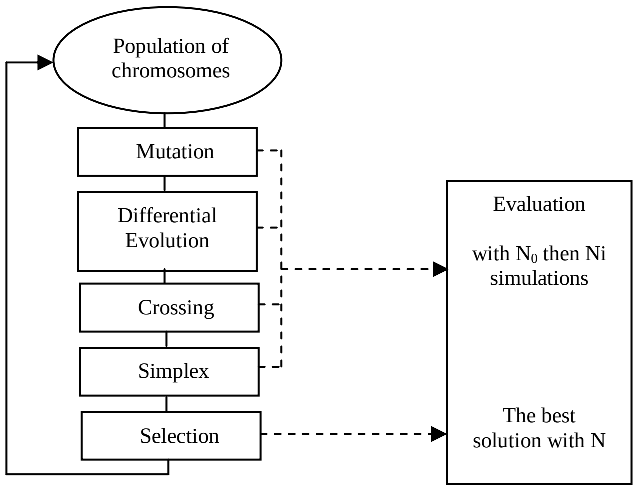


Figure 6. Principle of coupling

As showed in figure 6, this same principle could be applied to the Genetic Algorithms, the Differential Evolution and the Simplex after certain adaptations:

- The number of simulations performed during the rough estimation (N_0) and that necessary to required precision (N) are defined a priori by the user.
- The initial population of chromosomes (potential solutions) is evaluated with N0 simulations, then the best solution among this one (in average value) is revalued with N (by addition of $N-N_0$ simulations).
- During various loops of computation, each candidate i resulting from a mutation, a differential evolution, a crossing or a local research (simplex) is evaluated with N0 simulation then evaluated again with the value obtained from the

OCBA algorithm, limited to the N value (the re-valuation is effective only if $Ni > N_0$).

- The summation used by algorithm OCBA, in the calculation of the ratios, is updated at each evaluation, in order not to have to consider later on the previous solutions (not all memorized), and is re-initialized when a better solution appears, which then becomes the current optimal solution.

In addition, it appeared interesting not to require the maximum precision during all calculations but to increase the precision at the same time of the improvement of the population performance. So an evolution profile of the number of simulations, from the first to the last loop, was implemented in the tool.

4 APPLICATION CASE

The application case relates to the deployment and the renewal of a constellation of Earth observation satellites. The system average performance (number of operational satellites simultaneously in orbit) and the associated costs are evaluated during the entire mission (30 years) according to the characteristics of the satellites (reliability, lifespan, time of manufacturing…), of the launchers used (capacity, reliability, time of booking...) and of the selected strategy of renewal (decision criterion of replacement, spare satellite on ground or in orbit, anticipation of the end of lifetime...). Optimisation then consists in minimising the total cost of the constellation while respecting an objective of service availability.

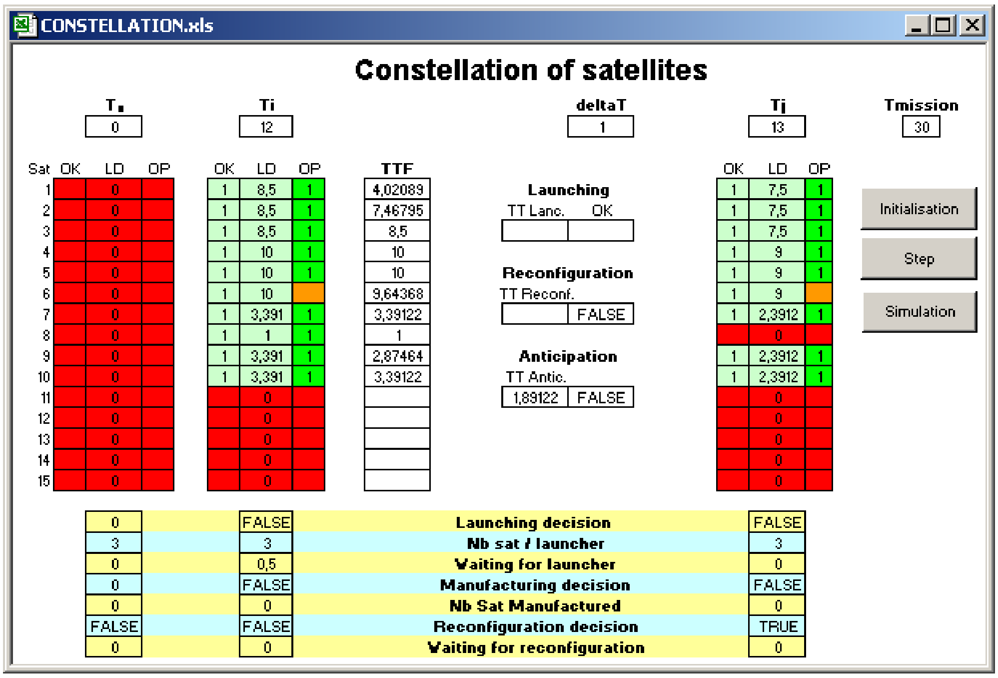


Figure 7. Constellation simulator

The recursive model presented of figure 7 allows simulating the deployment and the renewal of such a constellation. The satellites can fail (OK≠1) in a random way, according to an exponential law in this application (λ constant), or in a deterministic way at the end of the lifespan which is limited by the propellant capacity (LD: Life Duration). The constellation nominally consists of 7 simultaneously operational satellites. It is maintained by simple or multiple launches and the satellites can remain in orbit a certain time as a spare. Reconfiguration in the operational state (OP = 1) by ground command is then effective only after one month. A launching is decided as soon as the number of satellites functioning in orbit is lower than a required minimum number. The end of satellites lifetime is anticipated to avoid service interruptions. Launching is performed only after reservation duration of the launcher and after the satellites manufacturing if spare are not available on ground. The launcher can also fail. The assumptions considered are detailed hereafter (although the problem is real, the data presented are fictitious for reasons of confidentiality):

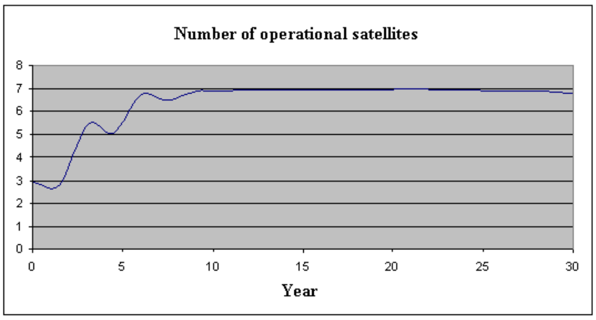
- Incomes of the constellation per year: 166 M€ with 7 satellites operational 66 M€ with 6 satellites and 0 M€ with less.
- The cost of the recurring satellites depends on their lifespan, of their reliability at the end of the lifetime and of their period of manufacture, according to the following formula:
  Satellite cost = 50 + 3 * (Lifespan - 5 years)^2 + 100 * (Reliability - 0,3) + 10 * (2 years - Period of manufacture)
- A type of launcher among five is selected for the first launch and another for the following. Each launcher has its particular characteristics:

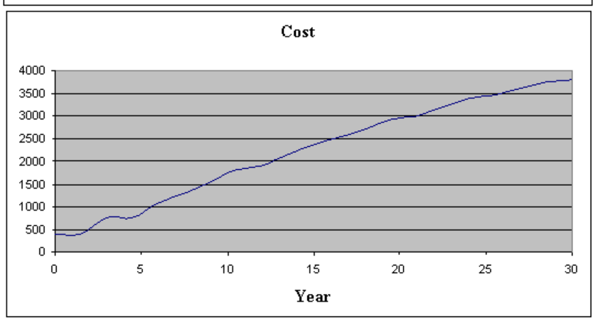
Type	1	2	3	4	5
Probability of success	0,95	0,97	0,98	0,9	0,93
Duration of reservation	0,6	0,5	0,3	1	0,8
Capacity max	6	3	2	1	5
Cost for :					
1 satellite	20	15	22	12	18
2 satellites	36	20	40		34
3 satellites	50	25			48
4 satellites	60				59
5 satellites	66				65
6 satellites	70				

Optimization seeks to maximize the profits over 30 years (incomes - cost) by modifying the following parameters:

• Type (1 to 5) and capacity (1 to 6) of the first launcher used,
• Type (1 to 5) and capacity (1 to 6) of the following launchers,
• Lifespan of the satellites (5 to 10 years),
• Reliability of the satellites at the end of the lifetime (0,3 to 0,8),
• Time of manufacturing for one satellite (0,5 to 2 years),
• Minimum number of satellites in orbit triggering a new launch (7 to 11),
• Manufacturing of ground spare after each launch: (true or false),
• Duration of maintain constellation in operational condition: (20 to 30 years).

The application case is evaluated in 1 minute approximately for 100 simulations of the 30 years mission with Pentium 4 (figure 8).





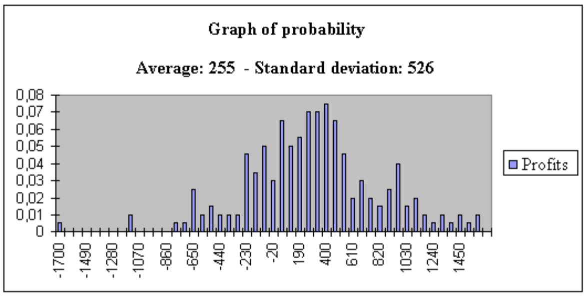


Figure 8. Results for a configuration of parameters

The search for an optimal configuration requires more than 2000 evaluations to solve problems similar by Markovian calculation. The total duration of the processing, without improvement of the coupling, should thus be approximately one week for 500 simulations per evaluation and one month for 2000 simulations.

5 RESULT OF OPTIMIZATION

Optimization was launched simultaneously on two computers, with one only using the improved coupling. The number of simulations per evaluation was between 50 and a value growing linearly from 50 to 2000 for the first, and equal to 500 for the second. 50 loops of calculation were performed by each of them. The standard tool adjustment was used (population of 50 chromosomes, improvement of the best chromosome by 50 steps of simplex, etc).

After less than 16 operating hours (one night), the first computer found the following solution evaluated with 2000 simulations. Figure 9 shows the result of this optimal configuration.

• Launcher type 2 with 3 satellites for the first launcher
• Launcher type 2 with 1 satellite for the following
• Satellite lifespan: 7,79 years
• Satellite reliability: 0,78
• Time of manufacturing: 0,51 years
• Minimum number of satellites in orbit triggering a new launch: 7
• Manufacturing of ground spare after each launch (but no spare in orbit)
• Duration of maintain constellation in operational condition: 29,57 years

After one week of calculation the second computer found a solution close although less powerful. The evaluation of each solution had been limited to 500 simulations instead of 2000 because of time constraints.

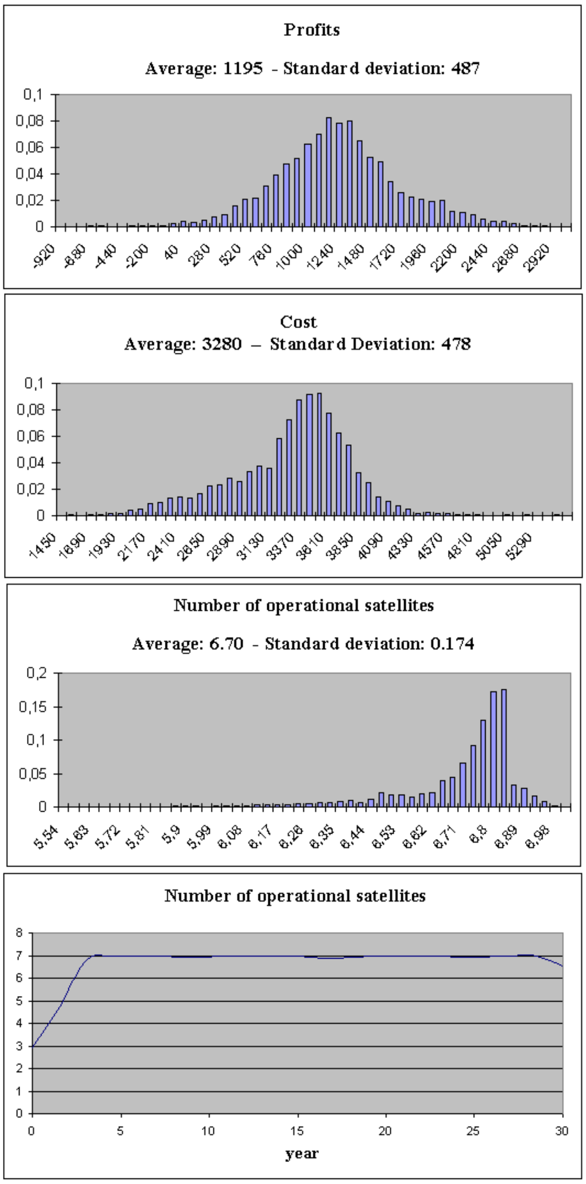


Figure 9. Result of the optimal configuration

The average number of simulations performed for each evaluation was 64 instead of 2000 (calculated by the tool throughout computation). The computing time was thus divided by 31.25 on this example compared to the same computation performed without improvement of the coupling.

Note: These tests highlighted a difficulty concerning the taking into account of constraints when optimization is performed starting from simulation results. Indeed, the industrial problems are often posed in terms of minimization of a cost with satisfaction of a constraint of performance (availability...) or maximization of a performance in a limited cost. But the cost and the performance are often opposed, and the optimum is generally located at edge of the constraint. However because of the variance of the results obtained by simulation, several evaluations of the same solution can strongly vary by considering that it respects or not the constraint. Also, the penalty associated with going beyond the constraint becomes difficult to adjust.

6 CONCLUSION

The original technique of recursive simulation modelling appears well adapted to the resolution of certain problems of systems with discrete states, possibly hybrid. It is an alternative to the use of other methods much more known which can sometimes lead to complex models difficult to validate. Based on the functionalities of a spreadsheet widely diffused, it allows developing and validating complex models within a time and at an extremely competitive cost. Such simulators can be developed in all fields of engineering (air transports, rail networks, telecommunications, energy...), in order to test the operational capacity of the systems and to optimize their characteristics and conditions of operating and maintenance, as of the preliminary phases of design.
The results obtained on various cases of application show that the advantage of the original coupling between optimization and Monte-Carlo simulation, proposed in this paper, is very significant in computing times (time divided by 30 approximately). This result allows using this type of processing, which offers many prospects, without waiting for much more powerful computers on the market.

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