

# Interest of a global optimization tool for reliability models adjustment and systems optimization

A. CABARBAYE & J. FAURE & R. LAULHERET

CNES, 18 av. Edouard Belin, 31401 Toulouse - France

A. CABARBAYE

Cab Innovation, 3 rue de la Coquille, 31500 Toulouse - France

**ABSTRACT:** This paper shows why a global optimization tool is essential for the adjustment of complex probabilistic model from feedback operational data. It is illustrated with several examples of reliability models (Bertholon, 3 phases) maintenance models (Generalized Renewal Process, Jack) and accelerated aging models (Arrhenius with Weibull, Basquin), with adjustment problem in the past and are performed here by a tool based on a hybrid technique combining genetic algorithms and non-linear simplex. It also shows the coupling possibilities of such optimization tool with assessment system model to optimize different parameters (period of preventive actions, depreciation duration, etc.).

## 1 INTRODUCTION

Different theoretical models are proposed to assess the system reliability, taking into account the maintenance, the specific conditions of use and the environment. Tools are available to adjust these models from feedback operational data but they may give wrong results, especially when the number of parameters in the model is greater than 2. The reason is simple. These tools implement local optimization methods (pseudo gradient, non linear simplex...) to make adjustments by the maximum likelihood method, while the model functions have multiple optima. Using a Global Optimizer can overcome these difficulties. Thus the GENCAB tool (Cabarbaye, 2003), based on a hybrid technique combining genetic algorithms and non-linear simplex, makes correct adjustments. So, this paper presents reliability models, maintenance models and accelerated aging models, with adjustment problem in the past. It also

shows the coupling possibilities of such optimization tool with assessment system model to optimise different parameters (period of preventive actions, depreciation duration, etc.).

## 2 THE GENCAB TOOL

Based on a hybrid method associating Genetic Algorithms, Differential Evolution and nonlinear Simplex (Nelder Mead algorithm), this generic tool under Excel® is illustrated by figures 1 and 2. Composed of various parameters (genes) of type real, integer or binary, the chromosomes are subjected to random mutation, crossings and differential evolutions (summation of a gene of chromosome with the difference between same genes of two other chromosomes). After selection, the best elements of the population can be improved at the local level by several steps of Simplex.

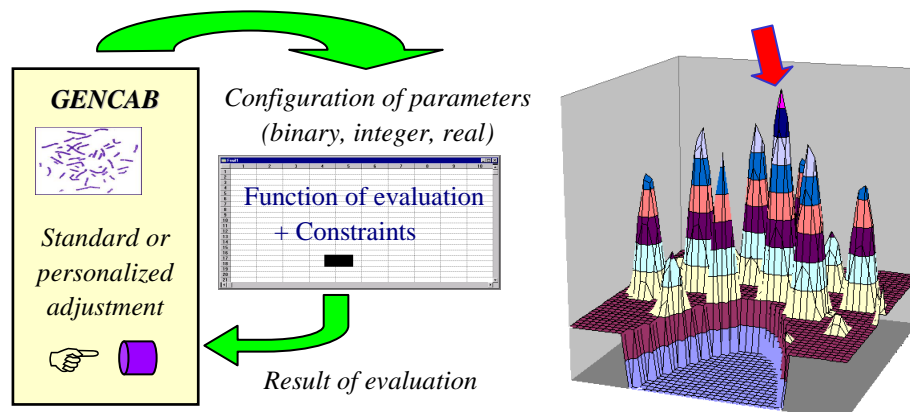


Figure 1. Principle of the tool

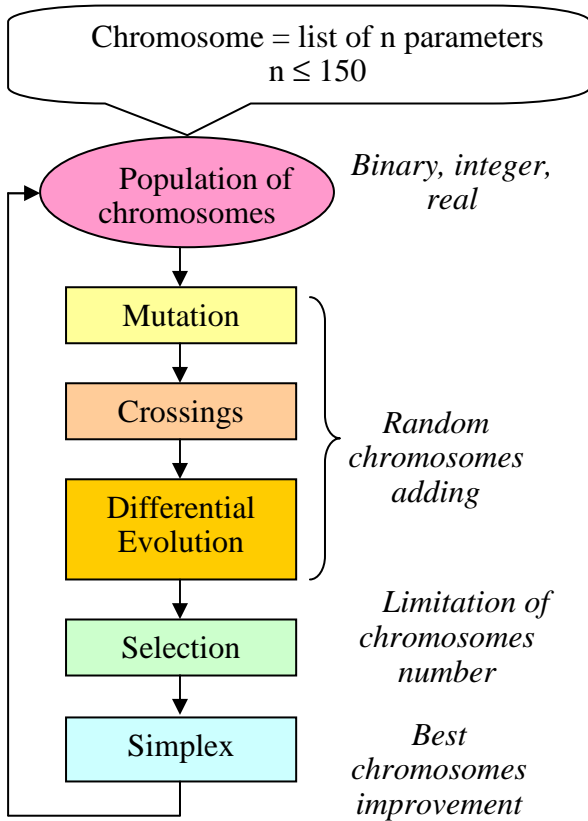


Figure 2. Algorithms

This hybridization of total (Genetic Algorithms) and local (Simplex) techniques, which can be possibly parameterized by the user, allows making the tool very efficient to find the global optimum and robust to the diversity of the problems addressed.

Thus the Differential Evolution will be generally more effective to treat a convex function but will present the disadvantage, for others, simultaneously to exploit the whole of genes. This feature makes it a valuable tool for the adjustment of complex probabilistic models and for the overall system trade-off among multiple parameters.

### 3 RELIABILITY MODELS ADJUSTMENT

#### 3.1 Bertholon model

Bertholon (Ziani, 2008) model combines an exponential and a Weibull for the overall second and third parts of the bathtub curve (occasional failures and wear). This model seems likely to be used in the future to characterize the reliability of electronic components whose integration should lead to more and more severe life limitations. The model consists of two blocks in series, one corresponding to an exponential law and the second to a Weibull law. Its reliability is expressed by equation 1.

$$R(t) = \exp(-\lambda t) * \exp(-[\max(0, t-T)/\eta]^\beta) \quad (1)$$

The occurrence of a fault can be simulated by the formula 2, under Excel, considering the lowest value simulated from these two models (obtained by reversing the distribution function from random value given by the ALEA function):

$$\text{MIN}(-\text{LN}[\text{ALEA}()]/\lambda ; T + \eta * (-\text{LN}[\text{ALEA}()])^{1/\beta}) \quad (2)$$

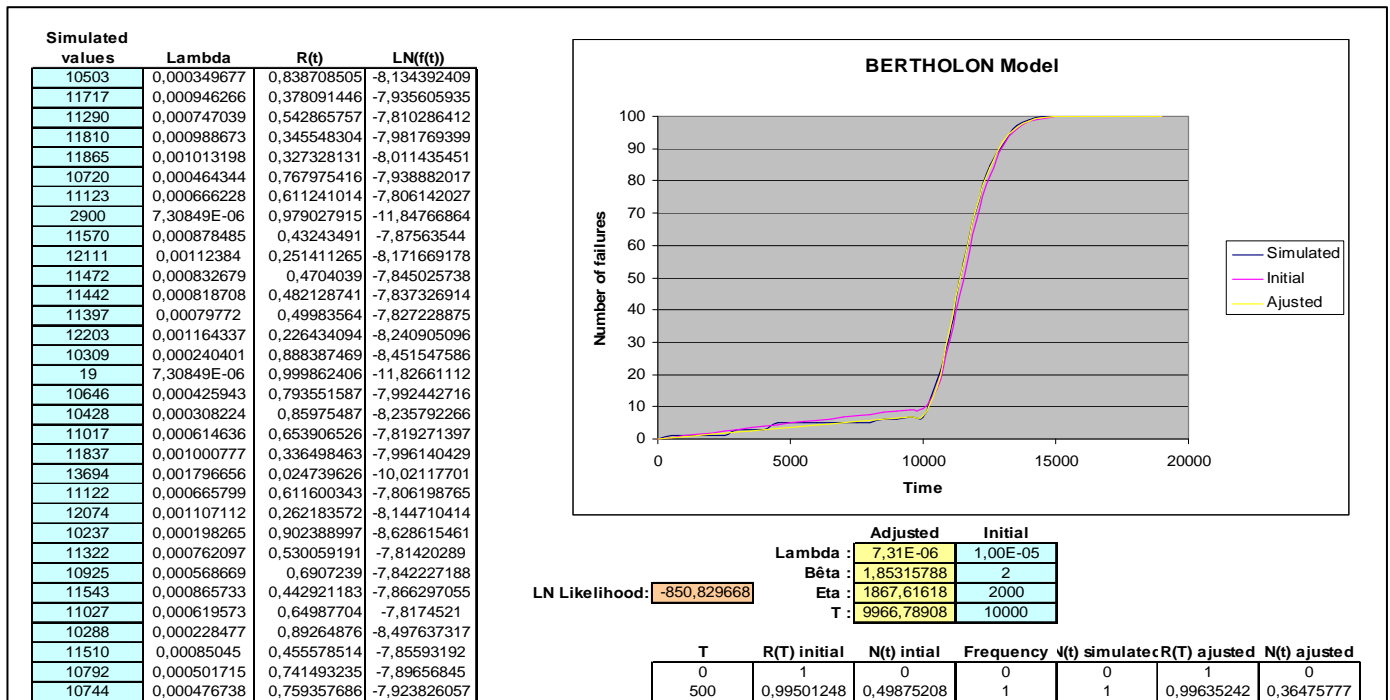


Figure 3. Simulation and fitting a Bertholon model (from 100 values)

The density of the Bertholon law can be expressed by equation 3:

$$f(t) = \lambda(t) * R(t) \quad (3)$$

$$\lambda(t) = \lambda + SI(t > T ; \beta * (t-T)^{\beta-1} / \eta^\beta ; 0)$$

$$R(t) = \exp(-\lambda t) * \exp(-[\max(0, t-T)/\eta]^\beta)$$

Figure 3 shows an adjustment of the Bertholon model made from a sample of 100 simulated values. The curves show the distribution functions of the theoretical model, the experimental model (simulated data) and the fitted model. We get approximately the values of model parameters used for simulation.

We remind that the method of maximum likelihood is to find the parameters of the theoretical law by maximizing the product of probability densities given by this law for the experimental data (or the sum of their logarithms). Performed by the optimization tool, the adjustment can be made from censored data by multiplying the product of densities by the product of the reliability values for censored data.

### 3.2 Model with 3 phases

Bertholon model can be generalized into a model to 7 parameters characterizing the three phases of the bathtub curve: a first Weibull law with  $\beta < 1$  for the phase of youth failure, an exponential law for the phase of occasional failures and a second Weibull

with  $\beta > 1$  for the wear phase. It corresponds to three blocks in series, the first is a Weibull, initiated at  $t = 0$  ( $\gamma = 0$ ) and limited to duration  $T_1$ , and the other two corresponding to the Bertholon model. The occurrence of failure can be simulated by the formula 4 under Excel.

$$(4) \quad T_i = \eta_1 * (-\text{LN}[\text{ALEA}()])^{1/\beta_1} \quad T = \text{si}(T_i < T_1; T_i; \text{MIN}(-\text{LN}[\text{ALEA}()]/\lambda; \gamma_2 + \eta_2 * (-\text{LN}[\text{ALEA}()])^{1/\beta_2})$$

The adjustment is the same way as above with the expression of the density given by the formula 5.

$$f(t) = \lambda(t) * R(t) \quad (5)$$

$$\lambda(t) = \text{si}(t < T_1; \beta_1 * t^{\beta_1-1} / \eta_1^{\beta_1}; 0) + \lambda + \text{si}(t > \gamma_2; \beta_2 * (t - \gamma_2)^{\beta_2-1} / \eta_2^{\beta_2}; 0)$$

$$R(t) = \exp(-[\min(t; T_1)/\eta_1]^{\beta_1}) * \exp(-\lambda t) * \exp(-[\max(0; t - \gamma_2)/\eta_2]^{\beta_2})$$

Again, we find approximately the values of model parameters used for simulation as shown in figure 4.

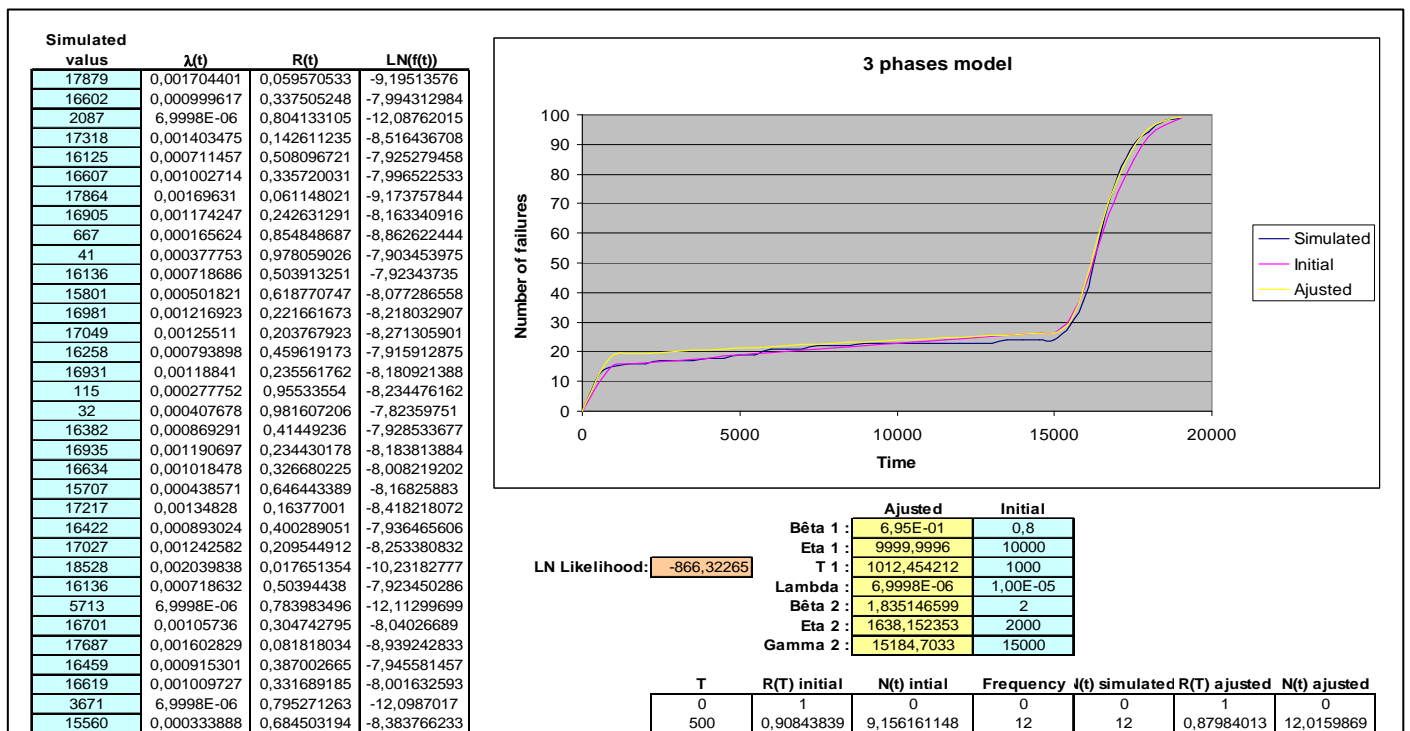


Figure 3. Simulation and fitting a 4 phases model (from 100 values)

## 4 MAINTENANCE MODELS ADJUSTMENT

### 4.1 Generalized Renewal Process

After corrective maintenance action the equipment can be

- as good as new
- as bad as old (in the state for his age)
- better than old but worse than new (in an intermediate state, excluding statements better than new or worse than his age).

Also 3 models have been proposed to model respectively the 3 types of maintenance:

- The Renewal Process (RP)
- The Non-Homogeneous Poisson Process (NHPP)
- The Generalized Renewal Process (GRP) Type 1 or 2.

In the GRP model type 1, the corrective maintenance has a rejuvenating effect proportional to the time elapsed since the previous maintenance. The virtual age ( $A_r$ ) of the equipment immediately after the  $r$ th maintenance action (at time  $t_r$ ) is given by the formula 6.

$$A_r = A_{r-1} + q^*(t_r - t_{r-1}) = A_{r-2} + q^*(t_{r-1} - t_{r-2}) + q^*(t_r - t_{r-1}) = q^* t_r \quad (6)$$

with  $q$  the rejuvenation factor .

$q = 0$ : complete rejuvenation (equivalent to a PR)

$q = 1$ : no rejuvenation (equivalent to a NHPP)

The probability that the equipment is down at  $t$ , knowing it was repaired at  $t_r$  is given by the formula 7.

$$\frac{[F(t)-F(t_r)]/R(t_r)}{R(t)/R(t_r)} = \frac{(1-R(t)-1+R(t_r))}{R(t_r)} = 1 - \frac{R(t)}{R(t_r)} \quad (7)$$

If the aging of equipment is modeled by a Weibull law, the corresponding distribution function is given by the formula 8.

$$F(t) = 1 - \exp\left[\left(\frac{qtr}{\sigma}\right)^\beta - \left(\frac{[qtr+(t-tr)]}{\sigma}\right)^\beta\right] \quad (8)$$

And the probability density is given by the formula 9.

$$f(t) = \beta \left(\frac{qtr+(t-tr)}{\sigma}\right)^{\beta-1} \frac{1}{\sigma^\beta} \exp\left[\left(\frac{qtr}{\sigma}\right)^\beta - \left(\frac{[qtr+(t-tr)]}{\sigma}\right)^\beta\right] \quad (9)$$

The GRP type 2 differs from type 1 by the fact that corrective maintenance led to a rejuvenation of the

equipment proportional to its virtual age. The virtual age is given by the formula 10.

$$A_r = q^*(A_{r-1} + t_r - t_{r-1}) = q^*(q^*(A_{r-2} + t_{r-1} - t_{r-2}) + t_r - t_{r-1}) = q^{r-1} * t_1 + q^{r-2} * (t_2 - t_1) + \dots + q^*(t_r - t_{r-1})$$

$$A_r = \sum_{i=1}^r q^{r-i+1} * (t_i - t_{i-1}) \quad (10)$$

The corresponding distribution function is given by the formula 11.

$$F(t) = 1 - \exp\left[\left(\frac{Ar}{\sigma}\right)^\beta - \left(\frac{[Ar+t-tr]}{\sigma}\right)^\beta\right] \quad (11)$$

And the probability density is given by the formula 12.

$$f(t) = \beta \left(\frac{Ar+t-tr}{\sigma}\right)^{\beta-1} \frac{1}{\sigma^\beta} \exp\left[\left(\frac{Ar}{\sigma}\right)^\beta - \left(\frac{[Ar+t-tr]}{\sigma}\right)^\beta\right] \quad (12)$$

Figure 5 shows an adjustment of the GRP model type 2 made from a sample of 200 simulated values. We get approximately the values of model parameters used for simulation. The curves show the results of simulation results (number of failures as a function of time for 1 and 2000 simulations).

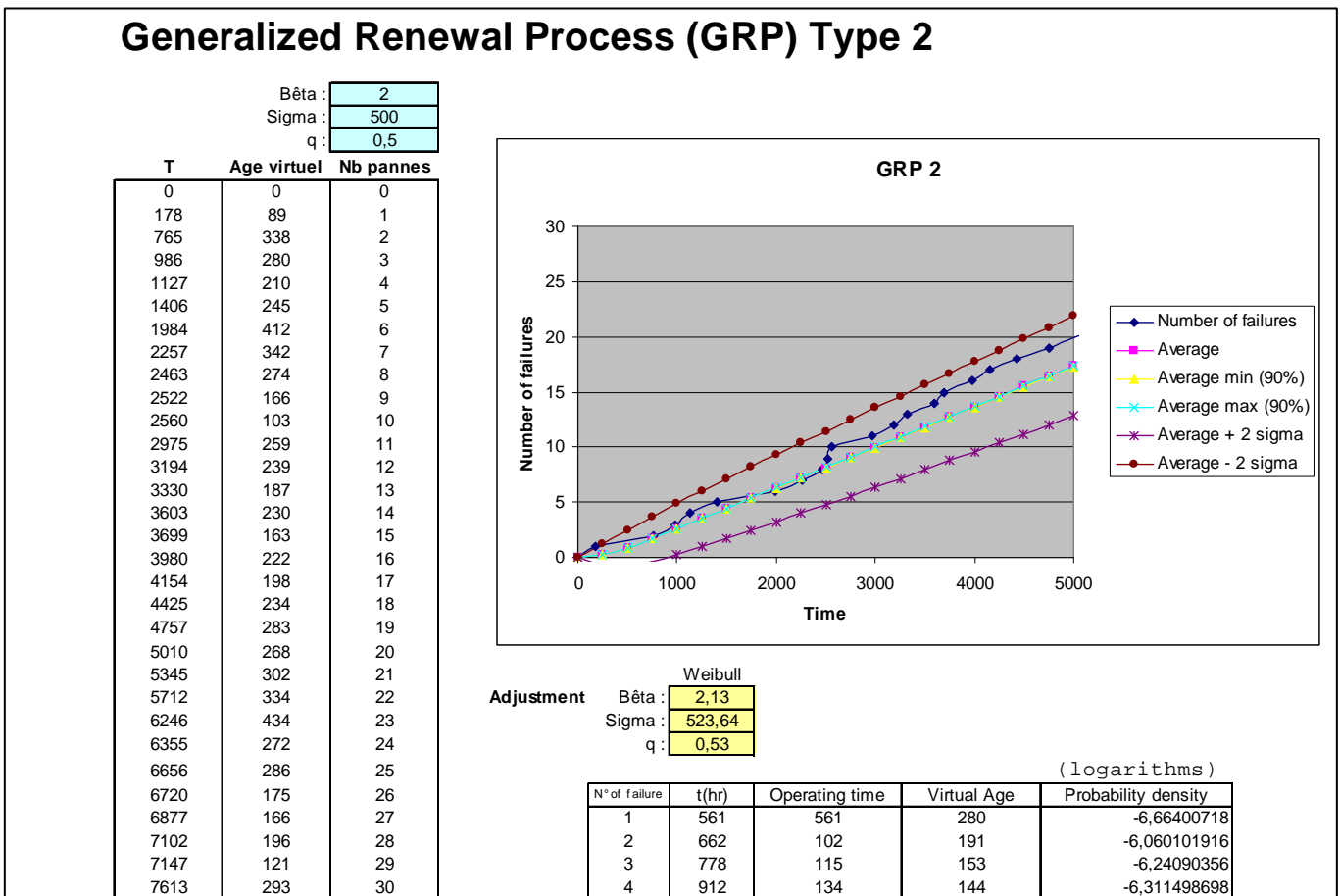


Figure 5. Simulation and fitting a GRP2 model (from 200 values)

#### 4.2 Jack Models

Jack (Jack, 1998) proposes two models of aging in which the preventive and corrective maintenance have an effect of rejuvenation. This effect is greater

in the case of a preventive maintenance (changing of several wear parts) than in the case of corrective maintenance (change of the one part down).

**Jack type 1 :**

- At the end of a corrective maintenance action, the virtual age of the equipment is equal to the one he had at the previous maintenance action (corrective or preventive) plus a proportion  $\rho_c$  of the time elapsed since.

- At the end of a preventive maintenance action, the virtual age of the equipment is equal to the one he had in the previous action of preventive maintenance, plus a proportion  $\rho_p$  of the time elapsed since.

**Jack type 2 :**

- At the end of a corrective maintenance action, the virtual age of the equipment is equal to the one he had just before this action multiplied by a proportion  $\rho_c$ .

- At the end of a preventive maintenance action, the virtual age of the equipment is equal to the one he had just before this action multiplied by a proportion  $\rho_p$ .

Similarly to the GRP model, the Jack type 2 model has been simulated by considering a periodic preventive maintenance, then it was adjusted from a sample of simulated values. Figure 6 shows the different results. Once again, we get approximately the values of parameters used for simulation.

It should be noted that the adjustment has always given a higher likelihood to that obtained with original parameters configuration used to generate the data sample.

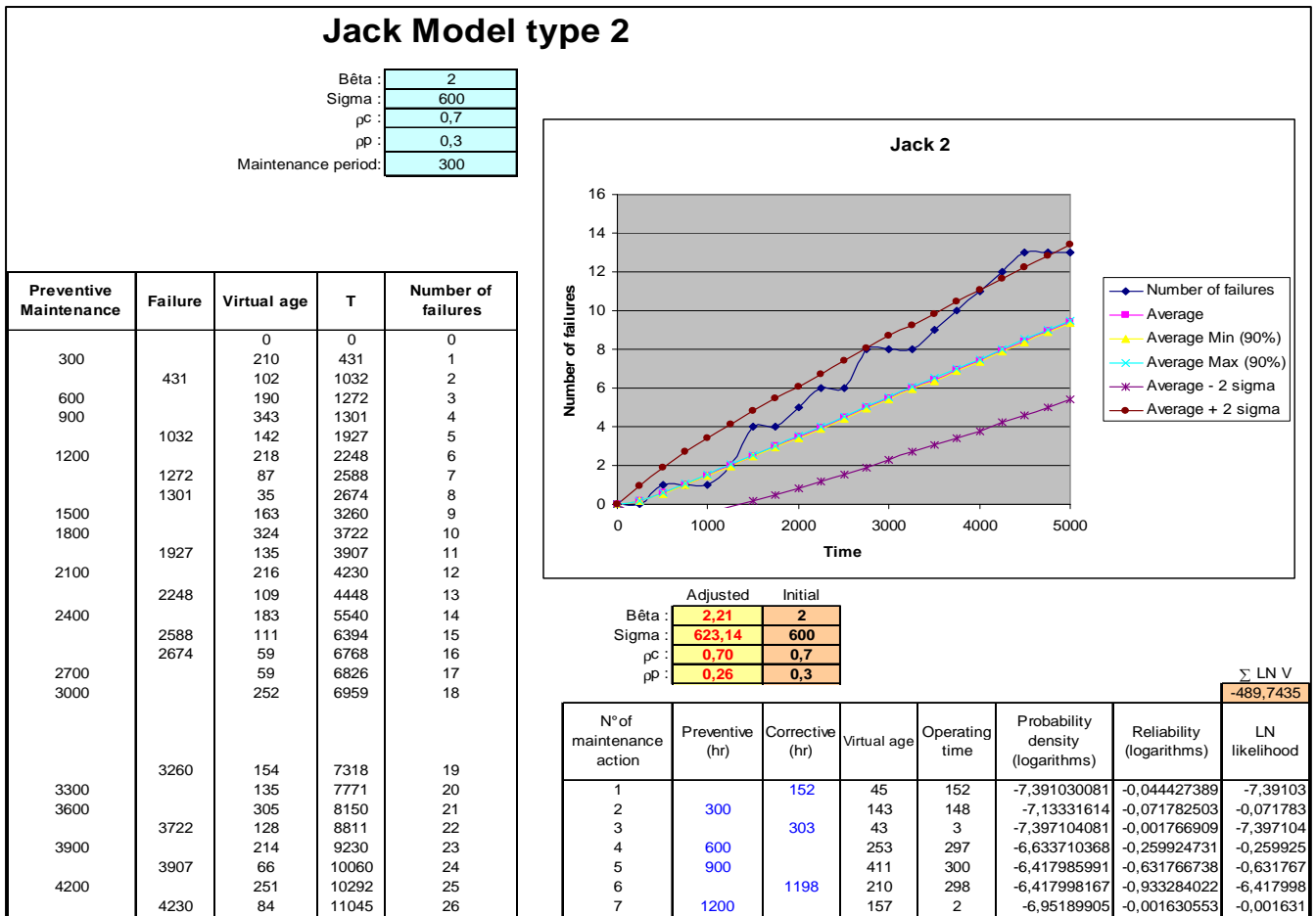


Figure 6. Simulation and fitting a Jack 2 model (from 200 values)

**5 ACCELERATED AGING MODELS ADJUSTMENT**

Accelerated aging models are useful to reduce the duration of reliability testing or assessing reliability under the conditions of use and the environment. But they can also be used to fit a model of reliability from operational data obtained under various conditions of use and the environment, as showed in the following examples.

**5.1 Arrhenius with Weibull model**

The Arrhenius acceleration factor (13) can make the correspondence between periods of operation under different temperatures.

$$FA = e^{\frac{Ea}{K} \left( \frac{1}{T_1} - \frac{1}{T_2} \right)} \quad (13)$$

With: T: absolute temperature in degrees Kelvin ( $^{\circ}K = ^{\circ}C + 273,15$ )  
 Ea: activation energy in eV (depending on the material and its mechanisms of damage varies between 0,2 and 1,1 eV)

K: Boltzmann constant (8,617385 10<sup>-5</sup> eV / ° K)  
 You may want to adjust a Weibull law from operational data obtained under various conditions of

temperature, without knowing the value of the activation energy of the product. Then you have to fit a model with 4 parameters as shown in Figure 7.

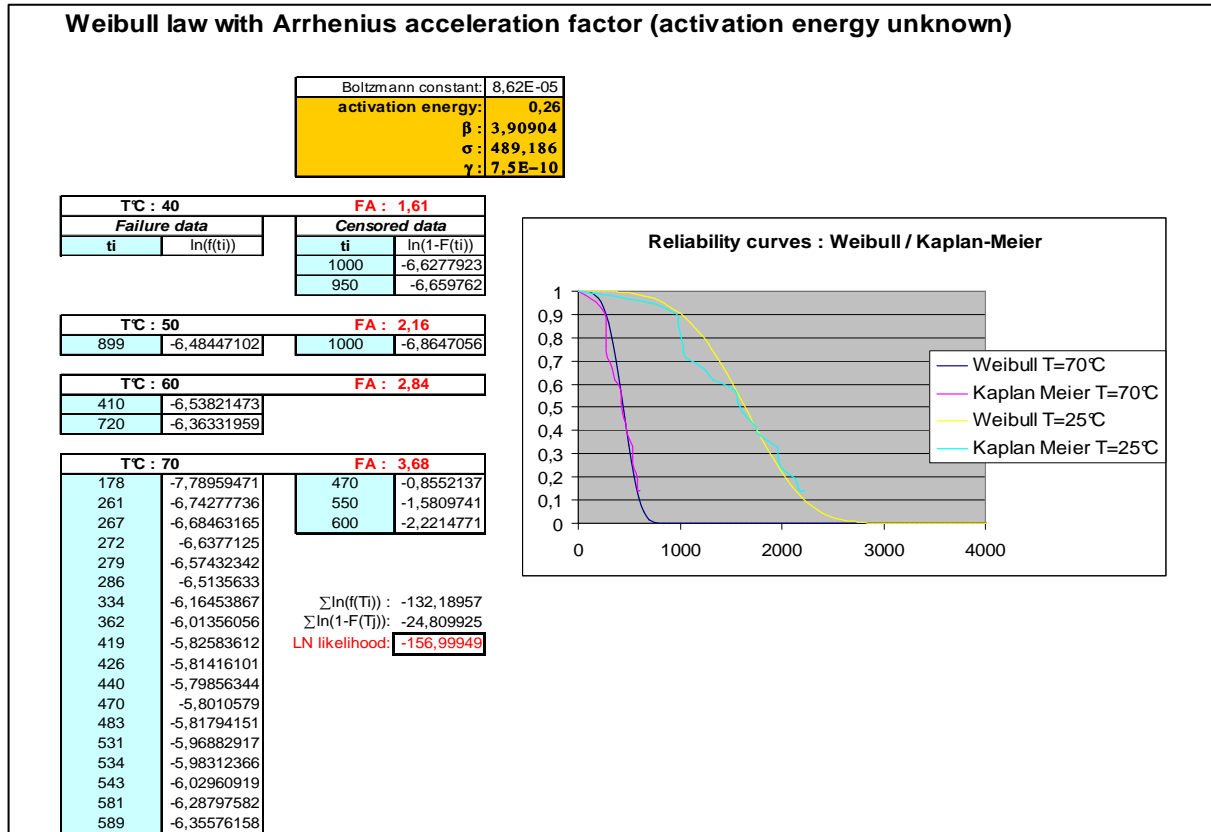


Figure 7. Fitting Weibull law with Arrhenius acceleration factor (activation energy unknown)

### 5.2 Fatigue model

Basquin's model (formula 14) is a model of fatigue acceleration making the correspondence between the numbers of cycles before failure for different values of constraint.

$$FA = N/N_0 = (\tau_0/\tau)^B \quad (14)$$

Moreover, experience shows that a significant dispersion exists between parts of the same batch. They are approximately following lognormal law based on the number of cycles.

From operational data obtained under various constraints, it is then possible to fit a lognormal law considering the coefficient B as additional parameter of optimization as shown in Figure 8.

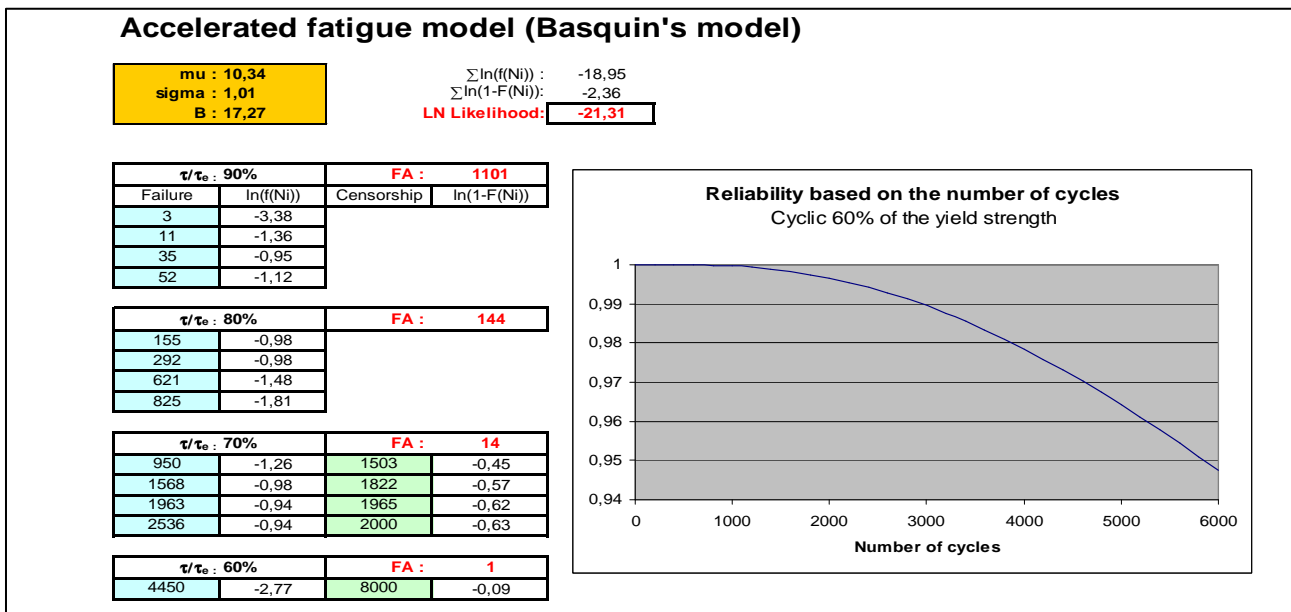


Figure 8. Fitting Lognormal law with Basquin' acceleration factor

## 6 OPTIMIZATION SYSTEM

### 6.1 Coupling between optimization tool and assessment model

The coupling between optimization tool and assessment model allows optimizing globally many parameters of architecture and operating system (level of redundancy, quality of components, maintenance actions duration, size of spare stock, period of preventive actions, depreciation duration, etc.) as shown in figure 9 (Faure, 2008).

The optimization can be based on a criterion and respecting various constraints such as availability

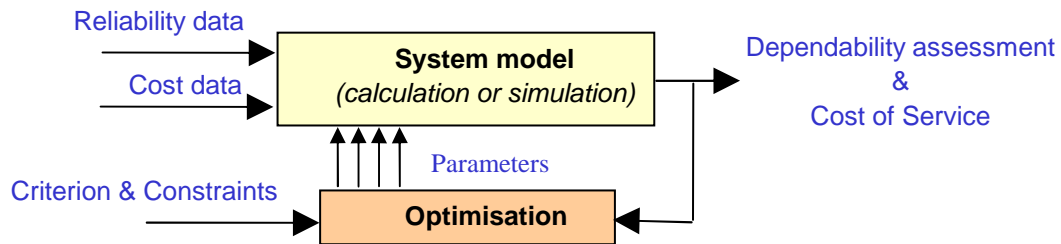


Figure 9. System optimisation

### 6.2 Optimization of preventive maintenance

In the case of equipment subject to wear, periodic maintenance may not be always optimal because preventive actions are too often at the beginning and not enough at the end of the process. Other strategies can be envisaged, such as those proposed below:

- Maintenance can be done to ensure the same level of risk of failure between two preventive maintenance actions..
- Preventive maintenance can be performed so as to make the average hourly cost of maintenance equal to a constant value during all the process.

Illustrated by figure 10, the first of these strategies has been the subject of optimization shown in Figure 11.

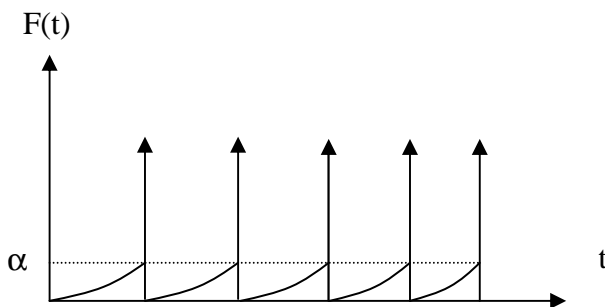


Figure 10. Preventive maintenance policy

Considering the average cost of preventive action ( $Cost_{preventive}$ ), the average cost of corrective action ( $Cost_{corrective}$ ), and the cost of equipment replacement ( $Cost_{replacement}$ ), the maintenance optimization

greater than 0,98 at minimum cost or maximum availability and cost less than 5000 k€ for example

The coupling between optimization and assessment model by Monte-Carlo simulation is particularly restrictive in computing time. That is why the tool GEN CAB has a particular algorithm (Cabarbaye, 2006) limiting to only the necessary number of simulations used to evaluate each solution (the overall computation time is roughly divided by 30 by this technique). The optimization of the maintenance (period of preventive actions, depreciation duration, etc.) can be made directly from a simulation model without having to use an analytical formula often complex or impossible to define. For example, optimization of preventive maintenance actions is the subject of the next section.

is to find the value  $\alpha$  of risk of failure and the duration of equipment depreciation ( $T_{depreciation}$ ) such that the average hourly cost, defined by formula 15, is minimized.

$$(N_p * Cost_{preventive} + N_c * Cost_{corrective} + Cost_{replacement}) / T_{depreciation} \quad (15)$$

$N_p$  and  $N_c$  are the respective average numbers of preventive and corrective maintenance actions performed during the period of depreciation of the equipment.

## 7 CONCLUSION

A global optimization tool is essential for the proper adjustment of probabilistic model a little complex from feedback operational data.

This kind of adjustment opens the door to many opportunities for optimization of architecture, maintenance or depreciation of the products.

Beyond the field of reliability, a global optimization tool is valuable to perform the overall trade-off of system engineering between multiple parameters.

The models presented in this paper were only chosen to illustrate the adjustment possibilities without analyse of their respective merits. Many others would have their place such as GEV (Generalized Extreme Value) or GPD (Generalized Pareto Distribution) laws used in the theory of extreme values, for example.

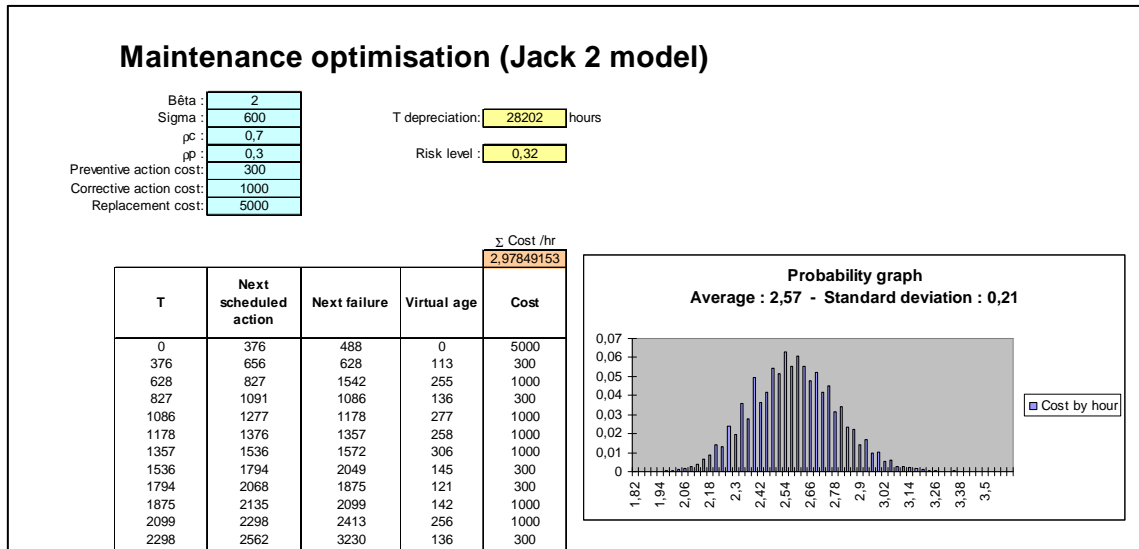


Figure 11. Optimization of preventive maintenance

## REFERENCES

- Jack, N.1998. Age-reduction models for imperfect maintenance. IMA Journal of Management Mathematics 9, pp. 347-354,
- Cabarbaye, A. 2003. Outil générique d'optimisation dans le domaine discret et/ou continu éventuellement stochastique - ROADEF'03 – Avignon.
- Cabarbaye, A. & Faure, J. & Laulheret, R. 2006. Couplage entre optimisation et simulation stochastique, ROADEF'06, Lille
- Faure, J. & Laulheret, R. & Cabarbaye, A. 2008. Is Optimized Design of Satellites Possible? ESREL, 22 - 25 September 2008, Valencia, Spain.
- Ziani, R. & Antoni, M. 2008. Modélisation du vieillissement des appareils de signalisation par le modèle de Bertholon et optimisation de la maintenance, lambda mu 16, 7-9 octobre 2008, Avignon, France.

GENCAB® is a tool of CAB INNOVATION Company  
[contact@cabinnovation.com](mailto:contact@cabinnovation.com)