Resolution of scheduling problems by using a generic optimisation software

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Abstract

This article evaluates the capabilities of a generic optimisation tool, based on a hybrid method coupling Genetic Algorithms and non-linear Simplex (algorithms of Nelder Mead), to solve various problems of scheduling met in space domain.

This tool appears relatively efficient and offers certain flexibility for the choice of the criteria (scheduling duration, cost with completion, etc). Its processing capability in the stochastic field (optimisation starting from results of Monte-Carlo simulation) allows getting a more robust planning with the risks, which can be improved or regenerated periodically throughout a project. This aptitude for simulation can be also used, within the framework of a risk analysis, to evaluate the impact of risks, in term of cost and time, in order to improve the project assessments or to justify the appropriateness of certain security measures.

Key words: scheduling, optimisation, genetic algorithms

1. Introduction

In project management or for piloting a production workshop, the scheduling is a difficult problem, which often covers economic stakes of first importance. It consists assigning to tasks resources and a temporal field of execution, by taking care to respect of certain constraints (Blasewicz and Ecker, 1993). It is in fact a combinatory optimisation problem in which a good solution must be found according to an evaluation criterion defined a priori by respecting certain constraints. Of course, this problem is not new, but its resolution becomes critical in a context of competitiveness and search for a better productivity, and the industrial repercussions of the many research carried out in this field are unusual (Pinedo, 1995). The passage of theoretical approaches to generic software packages is indeed not easy. The tools existing on the market are in the facts much more often used to improve the format of tasks schedules defined by the user that to truly define them. This is why the CNES evaluated the use of generic optimisation software, in order to facilitate the planning of development tasks of space projects. To analyse impacts of risks identified in the projects was also one of its concerns.

2. The generic optimization tool GENCAB

GENCAB Tool of CAB INNOVATION company is based on an hybrid method coupling Genetic Algorithms and non-linear Simplex (algorithms of Nelder Mead); This coupling having been selected to seek, in the most effective manner, optimal configuration of parameters (binary, integer or real) which maximizes or minimizes the result of an function defined by the user, without stopping with the first local optimum found. Functioning under Excel™ its general principle is illustrated by figure 1 [2].

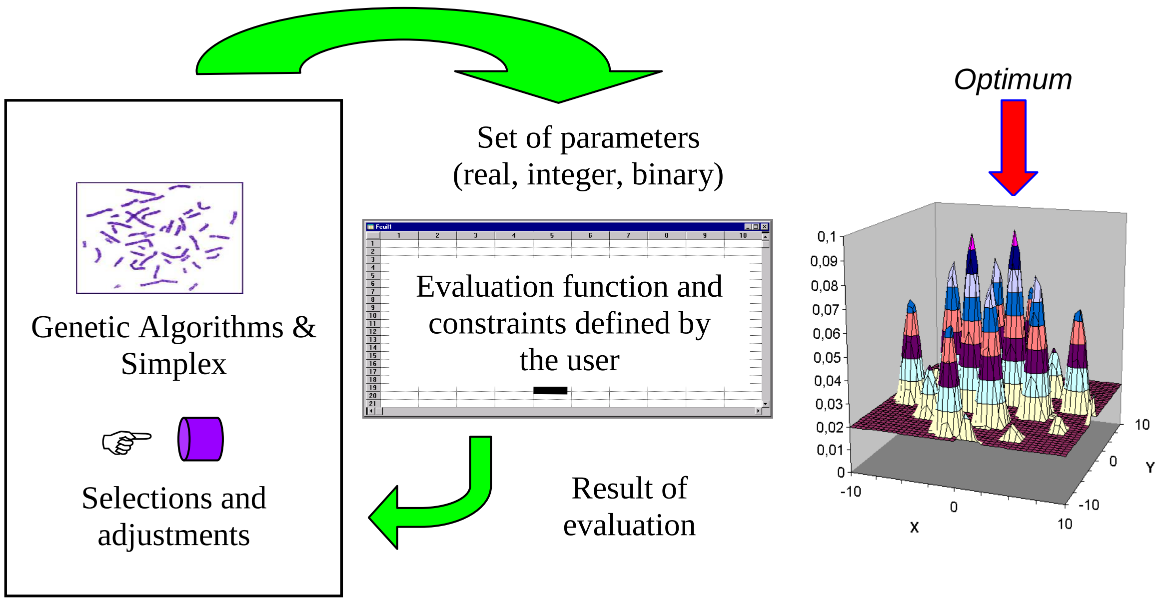

Figure 1. General principle of the generic tool for optimization GENCAB

The evaluation function is defined into an Excel sheet as well as possible constraints of the type A ≥ B between parameters or cells. Stochastic functions of evaluation can be also considered by using a second tool (SIMCAB) allowing to carry out optimisation starting from the results of a Monte-Carlo simulation (combination between average values and standard deviations).

3. Formalisation of the problem

Various tasks must be carried out by satisfying conditions of precedence and common resources such as those indicated in figure 2.

Tasks	Preceding tasks					Duration (days)	Common ressouces				
Task 1	10	6				150	2	5	8		
Task 2	1	3				55					
Task 3						80					
Task 4	3	6	2			125					
Task 5						215	8	9			

Figure 2 - Example of data input

In this example, task 1 start only after the end of tasks 10 and 6 and cannot be concomitant with the tasks 2, 5 and 8 for which common material or human resources are used. These resources do not need to be clarified and each condition can be only expressed one time (if task 1 cannot proceed during task 2, task 2 cannot proceed during task 1). The optimisation of scheduling consists in finding a configuration of dates Ti, beginning of each task i, satisfying a criterion, such as minimising the duration of the whole tasks, by respecting the constraints of priority and common resources.

4. Processing

By using the Excel functions directly, the problem was posed like in the table presented at figure 3, in which the value of the constraint of precedence is the sum of the unauthorized goings beyond (∑[ti-tj]), and that of the constraints of common resources, the sum of the unauthorized coverings; These two values having to be null so that the whole of the constraints is satisfied. The table then could be treated by the optimization tool to make disappear the nonsatisfied constraints and minimize the total duration of scheduling (in a few minutes for 25 tasks with Pentium 4 to 1 Ghz). A macro-function has been developed to generate the PERT diagram and to facilitate the reading of the results (figure 4).

	Tasks	Preceding tasks						Duration	Common resources						Beginning	End	Constraint of precedence	Constraint of resources
1	Task 1	10	6					150	2	5	8				565	715	11	0
2	Task 2	1	3					55							715	770	0	0
3	Task 3							80							244	324	0	0
4	Task 4	3	6	2				125							745	870	0	0
5	Task 5							215	8	9					92	307	0	0
6	Task 6	5						302							274	576	33	0
6	Task 7							459							274	733	0	0
8	Task 8							76	2	5					435	511	0	0
9	Task 9							89							336	425	0	0
10	Task 10							25							44	69	0	0
11	Task 11							46							755	801	0	0
12	Task 12							78	3	19					338	416	0	0
13	Task 13	9						13	10						518	531	0	0
14	Task 14							46							112	158	0	0
15	Task 15							78	11	10					546	624	0	0
16	Task 16	2	5					54							770	824	0	0
17	Task 17	2	1					69							833	902	0	0
18	Task 18							12							429	441	0	0
19	Task 19							78	8						610	688	0	0
20	Task 20							150							384	534	0	0
21	Task 21	3	6					29							825	854	0	0
22	Task 22	3	3					69	23						424	493	0	0
23	Task 23							31							244	275	0	0
24	Task 24							49	12	14					241	290	0	0
25	Task 25							53							928	981	0	0
															44	981	44	0

Global Duration: 937 days

Figure 3 - Table of treatment

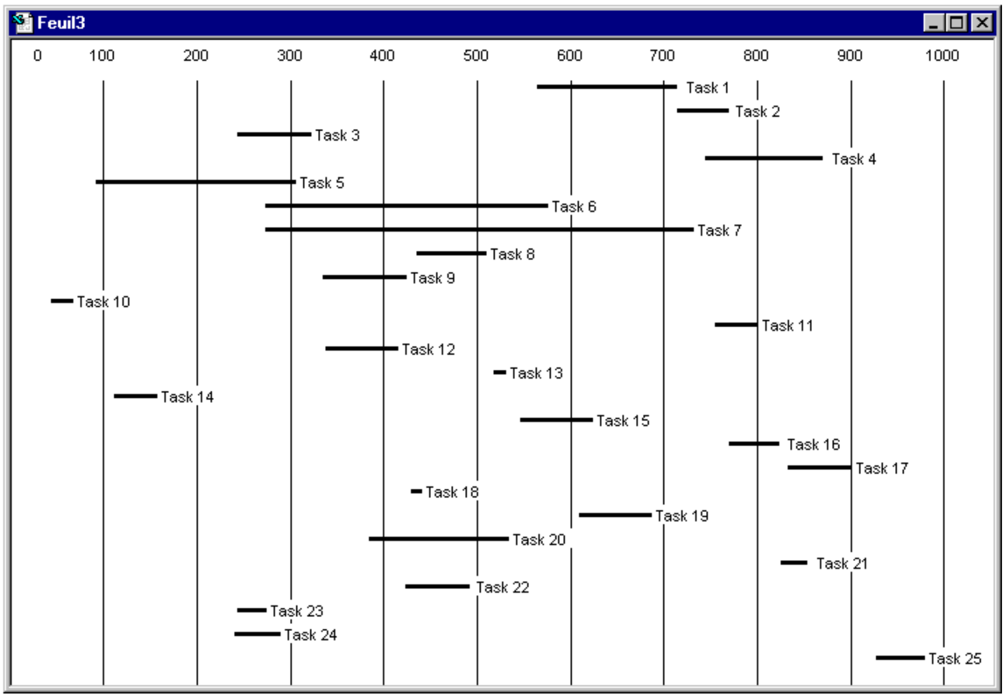

Figure 4 - PERT corresponding to the table

5. Minimization of the cost with completion

A space project having a relatively long duration and requiring heavy investments, it appears convenient to use a criterion more relevant than the minimisation of the scheduling duration. Also one chose to minimise the cost with completion by assigning to each task an original cost (different costs of provisioning can be replaced by a single cost at the beginning of task) and a cost proportional to his duration. These costs are updated by an interest rate until the completion date of the project. The table relating to this new problem is presented in figure 5.

Tasks	Preceding tasks						Duration	Initial cost (k Euro)	Cost / duration (k Euro)	cost with completion	Common resources						Beginn ing	End	Constraint of precedence	Constraint of resources
Task 1	10	6					150	10	1	233	2	5					573	723	0	0
Task 2	1	3					55	20	2	188							727	782	0	0
Task 3							80	45	3	427							114	194	0	0
Task 4	3	6					125	12	4	758							565	690	0	0
Task 5							215	50	25	7898	8	9					46	261	0	0
Task 6	5						302	4	32	13768							262	564	0	0
Task 7							459	5	2	1264							262	721	0	0
Task 8							76	78	3	425	2	5					782	858	0	0
Task 9							89	45	4	593							338	427	0	0
Task 10							25	13	2	95							138	163	0	0
Task 11							46	2	7	491							589	635	0	0
Task 12							78	6	3	360	3						292	370	0	0
Task 13	9						13	3	8	163	10						661	674	0	0
Task 14							46	7	9	637							541	587	0	0
Task 15							78	89	4	585	11	10					342	420	0	0
Task 16	2	5					54	2	3	248							803	857	0	0
Task 17	2	1					69	4	7	733							787	856	0	0
Task 18							12	6	12	227							823	835	0	0
Task 19							78	78	1	218	8						359	437	0	0
Task 20							150	2	1	224							491	641	0	0
Task 21	3	6					29	3	3	136							697	726	0	0
Task 22	3	3					69	5	8	839	23						440	509	0	0
Task 23							31	7	9	435							228	259	0	0
Task 24							49	6	50	3724	12	14					612	661	0	0
Task 25							53	45	7	619							384	437	0	0
										35288								858	0	0

Interest rate: 0,0005

Figure 5 - Minimization of the cost with completion

As previously, the optimisation tool could treat the table. Although the duration of each evaluation is a little longer, because of the addition of the columns of costs, the research of optimum is more effective. Indeed each variation of starting date of task has an impact on the final result, which was not the case previously (limited to the only tasks placed temporarily on the critical path).

6. Robust scheduling with the risks

In addition to the reliability analyses, risks analyses are carried out on space projects to identify the risks of delay increase or overcost. So, risk reduction actions are taken concerning each elementary task. Among these actions, the choice of a more robust scheduling can allow to mitigate some delays. Also we chose to add random variables to our table to characterise the duration and the cost of the tasks with associated risks (the tool proposes a score of statistical laws). Optimisation is then carried out starting from a result of Monte-Carlo simulation, which in the case of figure 6 corresponds to the average value of the cost with completion evaluated with 2 sigma. Different from the precedent with a bigger margin, scheduling result minimises the average cost of the whole simulated cases. The duration of calculation is then much longer and is roughly multiplied by the number of simulations (a few seconds by simulation with the computer used). This number is directly linked to the precision of the results (width of confidence interval inversely proportional to $\sqrt{n}$).

Tasks	Preceding tasks						Duration	Initial cost (k Euro)	Cost / duration (k Euro)	cost with completion	Common resources						Begin ning	End	Constraint of precedence	Constraint of resources
Task 1	10						175,7155	12,244314	1	290	2	5					511	687	0	0
Task 2	1	3					50,7388	20	2	186							718	769	0	0
Task 3							80	45	3	448							271	351	0	0
Task 4	3	6					125	12	4	803							758	883	0	0
Task 5							215	50	25	8382	8	9					53	268	0	0
Task 6	5						302	4	32	14613							288	590	0	0
Task 7							459	5	2	1341							288	747	0	0
Task 8							76	78	3	446	2	5					901	977	0	0
Task 9							89	45	4	627							403	492	0	0
Task 10							25	13	2	98							466	491	0	0
Task 11							46	2	7	521							667	713	0	0
Task 12							78	6	3	382	3						534	612	0	0
Task 13	9						13	3	8	172	10						662	675	0	0
Task 14							46	7	9	674							915	961	0	0
Task 15							78	89	4	610	11	10					528	606	0	0
Task 16	2	5					54	2	3	263							894	948	0	0
Task 17	2	1					69	4	7	778							833	902	0	0
Task 18							12	6	12	242							418	430	0	0
Task 19							78	78	1	212	8						751	829	0	0
Task 20							150	2	1	238							524	674	0	0
Task 21	3	6					29	3	3	144							660	689	0	0
Task 22	3	3					69	5	8	890	23						575	644	0	0
Task 23							31	7	9	459							770	801	0	0
Task 24							49	6	50	3953	12	14					450	499	0	0
Task 25							53	45	7	655							471	524	0	0
										37427								977	0	0

Interest rate: 0,0005 Cost with 2 sigm 37524 Total duration with 2 sigma: 924 days

Figure 6 - Optimization starting from simulation results

7. Extension of the problem

In the way presented previously, the evaluation was limited to 25 tasks. This relatively significant number (a project consists of macro-tasks which themselves can be broken up) could not be increased indefinitely because of limitations due to optimisation techniques used (between 30 and 50 different parameters according to the problem to be resolved). However a thorough analysis of the need showed that the majority of the tasks of a space project were inter dependent by constraints of precedence and not of common resources (those more often appear between various projects than within the same project). The majority of these tasks can start at the latest date with possible margin. In addition, some tasks must begin on fixed dates and others are constrained by a date of completion at the latest. Also, the problem was posed like the table presented in figure 7, in which the user can choose the parameters to be optimised (about thirty approximately to the maximum without limitation of the number of tasks). In this example, the beginning of tasks 3, 8, 10, 11, 14, 15 and 19 has to be optimised as well as the margins of tasks 1, 2 and 16 (these last would not have been inevitably null if optimisation had been carried out starting from a result of simulation to return scheduling robust to the risks as described in paragraph 5). Tasks 7, 9 and 12 begin on fixed dates and tasks 4, 6, 13, 17, 18, 20 to 25 on dates at the latest with margins. The tasks 19 and 24 are constrained by dates of completion at the latest.

Tasks	Preceding tasks						Duration	Initial cost (k Euro)	Cost / duration (k Euro)	cost with completion	Common resources						Latest date with margin	Beginning	End	End at the latest	Constraint of precedence	Constraint of resources	Constraint of dates of completion
Task 1	10						150	10	1	160	2	5					0	356	506		0	0	0
Task 2	1	3					50	20	2	120							0	506	556		0	0	0
Task 3							80	45	3	285								426	506		0	0	0
Task 4	3	6					125	12	4	520							2	620	747		0	0	0
Task 5							215	50	25	5425	8	9						0	215		0	0	0
Task 6	5						302	4	32	9700							1	265	568		0	0	0
Task 7							459	5	2	923								288	747		0	0	0
Task 8							76	78	3	306								546	622		0	0	0
Task 9							89	45	4	401								403	492		0	0	0
Task 10							25	13	2	63								331	356		0	0	0
Task 11							46	2	7	324								474	520		0	0	0
Task 12							78	6	3	240	3							534	612		0	0	0
Task 13	9						13	3	8	115	10						1	733	747		0	0	0
Task 14							46	7	9	421								535	581		0	0	0
Task 15							78	89	4	401	11	10						669	747		0	0	0
Task 16	2	5					54	2	3	164							0	556	610		0	0	0
Task 17	2	1					69	4	7	494							1	677	747		0	0	0
Task 18	16						12	6	12	150							0	610	622		0	0	0
Task 19	18						78	78	1	156	8							622	700	700	0	0	0
Task 20							150	2	1	154							2	595	747		0	0	0
Task 21	3	6					29	3	3	90							0	568	597		0	0	0
Task 22	3						69	5	8	557							0	528	597		0	0	0
Task 23	22	21					31	7	9	295							1	597	629		0	0	0
Task 24	23						49	6	50	2556	12	14					2	629	680	680	0	0	0
Task 25	24						53	45	7	430							2	692	747		0	0	0
Task 26	4	13	17	20	25		0	0	0	0								747	747		0	0	0
										24450								0	747		0	0	0

Interest rate: 0,0005

0 Parameter to be optimized

Figure 7 - Extension of the problem

The calculation of the beginning of the tasks starting with a date at the latest, with possible margin, is carried out simply by means of the elementary Excel functions by taking account the conditions of precedence. A fictive final task of null duration had to be added in the table to avoid circular problems of references (this task is moved until the end of scheduling during the optimization).

8. Risk analysis

In addition, the analysis of the need revealed a request of project managers to be able to evaluate the impact of risks, in term of cost and time, in order to improve their estimates or to justify the appropriateness of secure actions. A simple response to this need was proposed by duplicating the table and by replacing the beginning dates of tasks by the highest value among the dates obtained previously and those resulting from the satisfaction of the constraints of precedence and common resources (by means of the elementary Excel functions). The duration or cost modification relating to a task is translated then immediately, in this new table, by a modification of the whole results as shown in figure 8.

Tasks	Preceding tasks						Duration	Initial cost (k Euro)	Cost / duration (k Euro)	cost with completion	Common resources						Latest date with margin	Beginning	End	End at the latest	Constraint of precedence	Constraint of resources	Constraint of dates of completion
Task 1	10						150	10	1	222	2	5					0	356	506		0	0	0
Task 2	1	3					50	20	2	166							0	506	556		0	0	0
Task 3							80	45	3	395								426	506		0	0	0
Task 4	3	6					125	12	4	728							2	620	747		0	0	0
Task 5							215	50	25	7474	8	9						0	215		0	0	0
Task 6	5						302	4	32	13065							1	265	568		0	0	0
Task 7							459	5	2	1196								288	747		0	0	0
Task 8							76	78	3	411								546	622		0	0	0
Task 9							89	45	4	559								403	492		0	0	0
Task 10							25	13	2	88								331	356		0	0	0
Task 11							46	2	7	465								474	520		0	0	0
Task 12							78	6	3	340	3							534	612		0	0	0
Task 13	9						13	3	8	165	10						1	733	747		0	0	0
Task 14							46	7	9	602								535	581		0	0	0
Task 15							78	89	4	537	11	10						669	747		0	0	0
Task 16	2	5					108	2	3	460							0	556	664		0	0	0
Task 17	2	1					69	4	7	704							1	677	747		0	0	0
Task 18	16						12	6	12	215							0	664	676		0	0	0
Task 19	18						78	78	1	192	8							676	754	700	0	0	54
Task 20							150	2	1	215							2	595	747		0	0	0
Task 21	3	6					29	3	3	129							0	568	597		0	0	0
Task 22	3						69	5	8	794							0	528	597		0	0	0
Task 23	22	21					31	7	9	423							1	597	629		0	0	0
Task 24	23						49	6	50	3664	12	14					2	629	680	680	0	0	0
Task 25	24						53	45	7	598							2	692	747		0	0	0
Task 26	4	13	17	20	25		0	0	0	0								747	747		0	0	0
										33807								0	754		0	0	54

Figure 8 - 8. Risk analysis

In this example, the doubling of task 16 duration and the multiplication by 5 of its original cost, compared to figure 7, involve a light degradation of the total results and the non-observance of a constraint of date of completion at the latest (task 19). The tool capability of simulation can be also used to obtain the results in the form of statistics like those showed in figure 9 (the duration and the original cost of task 16 are modelled in this example by uniform laws). In the same way, all the risks identified can be simulated simultaneously, to improve the global estmations of cost and time.

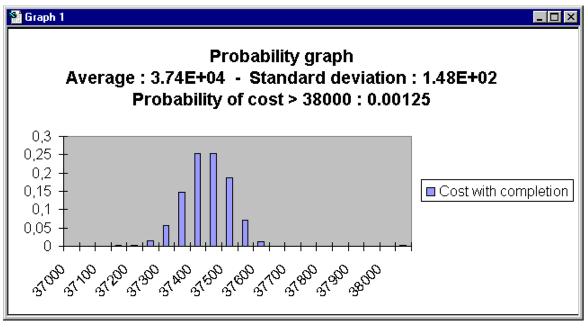

Figure 9 - Statistical results

Conclusion

The use of the GENCAB generic tool for optimisation proved efficient to solve problems of scheduling, particular because of the flexibility offered to the user for the choice of the criteria (the simultaneous use of a common resource for a limited number of tasks can for example be considered).

Its processing capability in the stochastic field allows to obtain a more robust planning with the risks, which can be improved or regenerated periodically throughout a project life by fixing the various parameters and variables already carried out.

This aptitude for simulation can be also used, within the framework of an risks analysis, to evaluate the impact of risks, in term of cost and time, in order to improves the project assessments or to justify the appropriateness of securisation actions.

However, the methods of optimisation used by the tool (genetic algorithms and non-linear simplex), do not allow guaranteeing the optimality of the solutions obtained. The user thus preserves his role of analyst and can always ask the tool to try to improve a known solution.

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