Optimization of an electric propulsion system

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Abstract : The purpose of this article is to provide an effective method to select the elements constituting an electric propulsion system among a database to maximize its efficiency and increase the performances of the aircraft. A calculation of the overall efficiency of the propulsion system is first proposed, taking into account the characteristics of the different elements. Then, an optimization method is studied. Finally, the proposed method is compared with a common method coming from model aircrafts community, in order to verify its accuracy.

1 Introduction

Too often, the performances of small UAVs in terms of payload or range, are much lower than expected, which sometimes jeopardizes projects of promising concept drones by lack of thrust for take-off. This performances lack is most of the time due to the propulsion system efficiency. Indeed, the Choice of the elements constituting an electric propulsion system is usually done using empirical formulas coming from model aircrafts community, based on empirical models of propellers such as ABBOTT/YOUNG or BOUCHER 's formulas :

$$W_{mechanical} = P \times D^4 \times R^3 \times 5.33 \times 10^{-15}$$

$$W_{mechanical} = K \times {}^{P}\!/_{12} \times \left({}^{D}\!/_{12}\right)^4 \times \left({}^{R}\!/_{1000}\right)^3 \quad (1)$$

where W is the mechanical power (watts), P the pitch (inches), D the diameter (inches), R the rotation speed (RPM) and K an adjustment parameter depending on the propeller type. These approximations only give an order of magnitude hardly more accurate than 20%. It does not take into account the propeller efficiency which depends a lot on the airspeed velocity. That is why a new method of optimization of the constituting components of a propulsion system is proposed in this article.

2 Efficiency definition

The aim of the powerplant optimisation is to minimise the energy consumption in order to either increase the flight range for a given inboard energy capacity, or to increase the payload capacity for a given maximum take-off mass. The study will be sliced into two parts :

Dynamic applications : (airplane in cruise, climb...) The optimised parameter is the classical power efficiency of the system, usually noted : η where

$$\eta = Power_{required}/Power_{consumed}$$

Static applications : (helicopter, airplane take-off beginning...) For these applications, the powerplant command is the thrust force required. Therefore, the optimised parameter η is defined as :

$$\eta = Thrust_{required}/Power_{consumed}$$

In both cases, the total efficiency depends on the brushless engine efficiency, and on the propeller efficiency :

$$\eta_{dynamic} = \eta_{Brushless} \times \eta_{propeller_{dynamic}}$$

$$\begin{aligned}\eta_{static} &= \frac{Thrust_{required}}{Power_{consumed}} \\ &= \frac{Thrust_{required}}{Power_{mechanical}} \times \frac{Power_{mechanical}}{Power_{consumed}} \\ &= \eta_{brushless} \times \eta_{propeller_{static}}\end{aligned}$$

2.1 Brushless engine

A brushless engine can be modelled as shown on the figure 1.

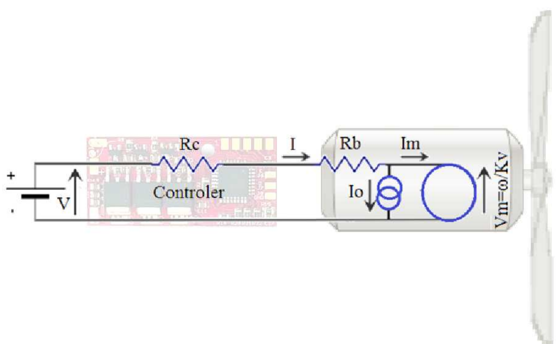

Figure 1: Brushless electrical motor diagram

There are two main electromechanical motor constants [1] :

$$RPM = K_v V_m, \text{ and : } Torque = K_m I_m$$
$$\text{with : } Kv = 1/K_m$$

where V_m is the electromotive tension, I_m is the engine useful current K_m the motor size constant, and K_v is the motor velocity constant. The efficiency is defined as :

$$\eta_{motor} = P_{meca}/P_{batt}$$

$$\text{where : } P_{batt} = VI, \text{ with : } I = I_m + I_0$$
$$\text{and : } P_{meca} = V_m I_m, \text{ with : } V_m = V - (R_c + R_b)I$$

so :

$$\eta_{motor} = \frac{[V - (R_c + R_b) \times (I_m + I_0)] \times I_m}{V \times (I_m + I_0)}$$

Usually, manufacturers provide the Kv of their engines so :

$$Torque = K_m I_m \text{ becomes } I_m = K_v Torque$$

then :

$$\eta_{motor} = \frac{[V - (R_c + R_b) \times (K_v \times Torque + I_0)]}{\mathrm{V \times (K_v \times Torque + I_0)}} \times K_v \times Torque \quad (2)$$

2.2 Propeller

2.2.1 Dynamic application

According to Rogers, David F. [2], the efficiency of a full scale fixed pitch propeller is a function of the advance ratio :

$$J = V/(\omega D) \quad (3)$$

where V is the infinite airspeed, ω is the propeller rotation speed, and D is the propeller diameter. A test with the propeller model 8x7 sport of the manufacturer APC Propellers seams to confirm this trend as well for model propellers for a rotation speed range of 5000 RPM as it can be seen on figure 2. But model propellers efficiency is much more sensitive to rotation speed variation as it modifies much more the Reynold number than for full scale propellers [3], as it can be noticed for the same propeller model 8x7 sport of APC Propellers but over a much wider range of rotation speed, as shown on the figure 3.

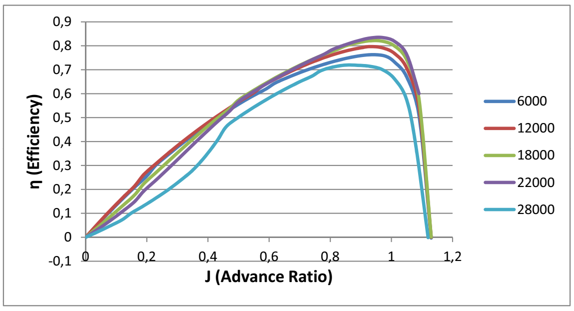

Figure 3: Efficiency function of the Advance Ratio for large range of RPM

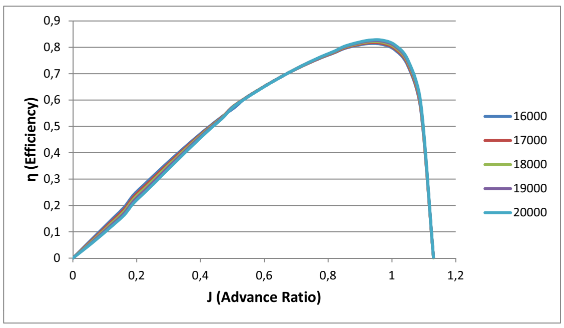

Figure 2: Efficiency function of the Advance Ratio for low range of RPM

In light of these two observations, and in order to do the optimisation efficiently, it is proposed to split the propellers rotation speed range in several portions over which the efficiency variations are negligible, and which are each related to an efficiency trend line.

Therefore the propeller efficiency can be estimated as :

$$\eta_{propeller} = f(J) \quad (4)$$

The function f seems to depend a lot on the propeller geometry, and an approximation generalized to all propellers does not seem relevant. Therefore, the function of each propeller will be downloaded, and the optimization process will select the best propeller

2.2.2 Static application

In static application, the efficiency still depends on the rotational velocity as it can be seen on figure 4.

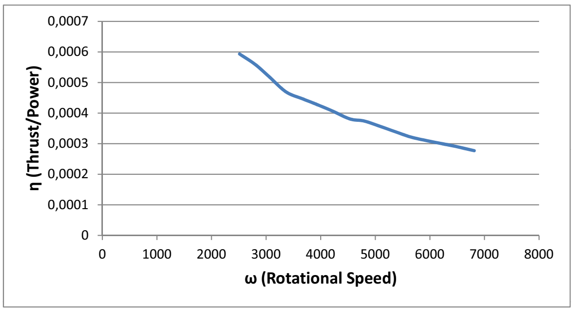

Figure 4: Efficiency function of the rotation speed

Therefore the propeller efficiency can be estimated as :

$$\eta_{propeller} = f'(\omega) \quad (5)$$

Note : The propellers caracteristics seem to vary depending on the source where they are found. It is adviced to collect these caracteristics from a unique source for the optimization in order not to be dependent on the test conditions. For instance, the garnished data base made by the University of Illinois at Urbana-Champaign Applied Aerodynamic Group may be a good one [4].

2.3 Rotation speed

2.3.1 Dynamic application

According to Waine Jonshon [5] :

$$T = 2\rho Av(V+v), \text{ and } P_{air} = T(V+v)$$

or :

$$P_{usefull} = \eta_{propeller}(J)P_{air}$$

so :

$$P_{usefull} = \eta_{propeller}(J)2\rho Av(V+v)^2$$

The propeller pitch p is defined such that

$$V + v = \omega p$$

which combined with (3) gives $v = \omega p - J\omega D$ then :

$$P_{usefull} = {}_{propeller}(J)2\rho A(p - JD)p^2\omega^3$$

which is of the form :

$$P_{usefull} = f(J)\omega^3$$

A test with the propeller model 8x7 sport of APC Propellers seams to confirm this trend for model propeller for a rotation speed range of 5000 RPM as it can be seen on figure 5.

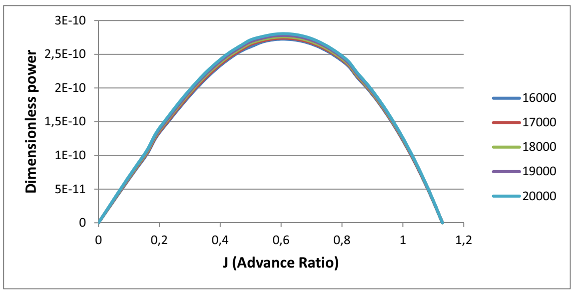

Figure 5: Power made dimentionless in ω^3 function of the Advance Ratio for low range of RPM

The earlier observation regarding the Reynold number applies [3], as it can be noticed for the same propeller model 8x7 Sport of APC Propellers over a much wider range of rotation speed, as shown on the figure 6.

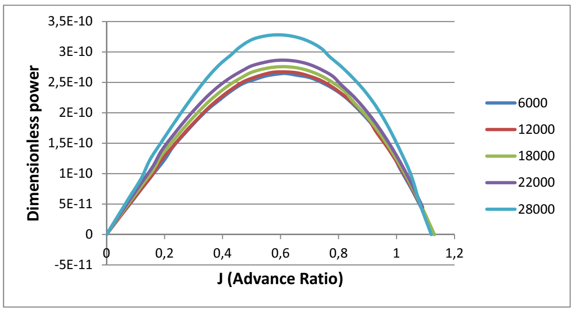

Figure 6: Power made dimentionless in ω^3 function of the Advance Ratio for large range of RPM

Therefore, as done before, it is proposed to split the propellers rotation speed range in several portions over which the power variations are negligible.

The obtained trend line is of the form :

$$f(J) = aJ^2 + bJ$$

so :

$$P_{usefull} = (a(V/(\omega D))^2 + bV/(\omega D))\omega^3$$

$$bV/D\omega^2 + a(V/D)^2\omega - P_{usefull} = 0$$

which gives :

$$\omega = \frac{-a(V/D)^2 + \sqrt{(a(V/D)^2)^2 + 4bV/DP_{usefull}}}{2bV/D} \tag{6}$$

Note : The chosen solution is the higher of the two. Indeed, it represents the one with the higher power efficiency (cf, power efficiency graph).

2.3.2 Static application

Usually, it is assumed that in static, the produced thrust follows the relation :

$$Thrust_{static} = K\omega^2$$

where K is a constant. This relation comes from the Thrust coefficient definition :

$$C_T = T/\rho A(\omega R)^2$$

giving :

$$T = (C_T\rho AR^2)\omega^2$$

But C_T is sensitive to the reynold number ref2, and as it can be seen on figure 7, the approximation in ω^2 is not accurate.

It seems to be much more accurate to approximate the thrust with a function of the form :

$$Thrust = a\omega^2 + b\omega$$

and then the rotation speed ω can be estimated :

$$\omega = \frac{-b + \sqrt{b^2 + 4aThrust}}{2a} \tag{7}$$

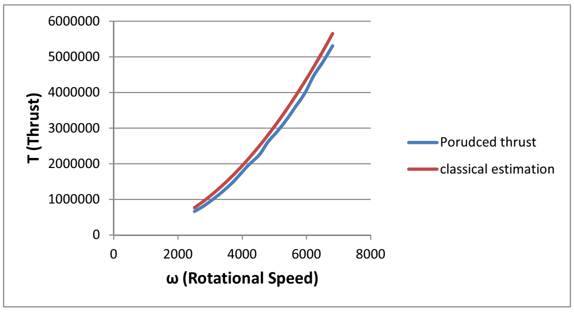

Figure 7: Thrust function of the rotation speed

2.4 Torque computation

A function relating the torque to the required power could be computed and made dimensionless. But in order to gain time, the torque is computed with the two characteristic functions already computed for the propeller.

2.4.1 Dynamic applications

We have : $P_{usefull} = \eta_{propeller} \times P_{mechanical}$, where $P_{mechanical} = Torque \times \omega$, and $\eta_{propeller} = f(J)$ so :

$$Torque = P_{mechanical}/\omega = P_{usefull}/(\omega f(J)) \tag{8}$$

2.4.2 Static applications

We have : $Thrust = \eta_{propeller} \times P_{mechanical}$ where $P_{mechanical} = Torque \times \omega$ and $\eta_{propeller} = f'(\omega)$ so :

$$Torque = P_{mechanical}/\omega = Thrust/(\omega f'(\omega)) \tag{9}$$

2.5 Total

Then the global efficiency can be computed.

$$\eta_{total} = \eta_{motor}\eta_{propeller}$$

2.5.1 Dynamic applications

Combining equations (4), (8) and (2), The expression of the global efficiency η_{total} is obtained :

$$\eta_{total} = \frac{[V - (R_c + R_b)(K_v \times P_{usefull}/(\omega \times f(J)) + I_0)]}{V \times (Kv \times P_{usefull}/(\omega \times f(J)) + I_0)} \times K_v \times P_{usefull}/(\omega \times f(J)) \times f(J) \tag{10}$$

with ω following equation (6).

2.5.2 Static applications

Combining equations , (5) (9) and (2), The expression of the global efficiency η_{total} is obtained :

$$\eta_{total} = \frac{[V - (R_c + R_b)(K_v \times Thrust/(\omega \times f'(\omega)) + I_0)]}{V \times (Kv \times Thrust/(\omega \times f'(\omega)) + I_0)} \times K_v \times Thrust/(\omega \times f'(\omega)) \times f'(\omega) \tag{11}$$

with ω following equation (7).

3 Optimisation method

The aim of the optimisation is to select the combination of a propeller, an engine and a brushless motor, from a data base, which provides the best performances.

Assigning an integer at each model of the components, the problem becomes an Integer Programming (IP) with three optimization parameters.

The optimal solution can be found by an exact method and the time required depends of the number of possible values of different optimization parameters.

Two kind of method can be used to the optimal solution :

1. Exact methods

   - The optimal solution is found.
   - The time required to find it is a linear function of the number of differents products and an exponential function of the number of parameters considered.

     These methods work well when the choice of the components is limited, but can be rapidely problematic when the components data base is bushy

   Furthermore, the optimization of propulsion can be integrated into an overall system optimization, in an obvious perspective of greatly improving the overall performances. In this case, the three integer parameters mentioned above are combined with other parameters, whole or real (eg engine position, **Aircraft Fuel Penalty** ...); and the problem becomes a mixed integer programming (MIP).

2. Stochastic methods :

   The performance function is rarely convex but generally has multiple local optimums. Therefore, a local method as the gradient cannot be used. In this case, only the stochastic methods can be used to find a good solution. Indeed, stochastic methods like heuristic or Meta heuristics have largely proved their effectiveness in finding global optima, although the optimality of the solutions cannot be guaranteed or demonstrated [6].

The software used for this study is based on genetic algorithms, differential evolution and non-linear simplex (Nelder Mead algorithm). This hybridization of global and local techniques allows the convergence to be accelerated and the tool to become robust to the variety of problems [7].

Developed by Cab Innovation under Excel, Gencab tool is illustrated on figure 8. Composed of various parameters (genes) of type real, integer or binary, the chromosomes are subjected to random mutation, crossings and differential evolutions (summation of a gene of chromosome with the difference between same genes of two other chromosomes). After selection, the best elements of the population can be improved at the local level by several steps of Simplex.

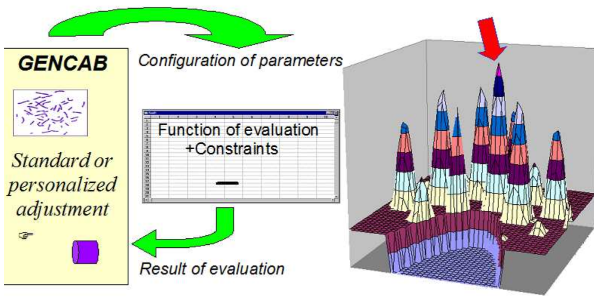

Figure 8: Principle of the tool

4 Problem implementation

The optimization settings : As it has been seen before, there are three settings to optimise. 3 intergers referring respectively to the engine model, the propeller model and controller model.

Note : To make the most effective optimization, it is advised to classify the different models according to one of their parameters, in particular the motors K_v, the propellers pitch, and the maximum current reachable by the controllers.

The optimization constraints : In the basic optimisation, there will be at least two constraints :

- The first is a limitation on the rotational speed : It must not exceed the highest engine rotational speed that can be figured out with the expression :

$$\omega \leq \omega_{maximum} = V \times K_v$$

- The second concerns the intensity. It should not exceed the maximum ones of the motor and of the controlor :

$$I_{maximum} \leq min(I_{controller_{maximum}}; I_{motor_{maximum}})$$

.

the optimization criterion : The aim of the optimisation is to optimize the global efficiency of the propulsion system. It can be done separatly using the formulas described above, or combined with a global aircraft optimization to obtain even higher performance.

5 Application example

The methodology proposed in this article has been tested on a simple case. It is proposed to optimize the electric propulsion system of an aircraft type trainer (wingspan : 1.1m, weight : 700g), whose characteristics are :

- Climb : $Airspeed = 10m/s$; $Power_{useful} = 50W$
- Cruise :$Airspeed = 21m/s$; $Power_{useful} = 35W$

The component of the propulsion system are Selected within a database of 10 propellers, 15 electrical motors, and 2 controllers.

The problem studied is a bit more complex than the basic one. Indeed, a third constraint is added, it concern the climb performance. It is required to provide the climb power withoud exceeding the intensity and rotation speed limitations.

To do so, a first computation is performed with the cruise conditions, giving the total efficiency η_{total}, which is minimised by the software, and the rotation speed and the intensity in cruise, which must respect the constraints; then a second computation is performed with the climb conditions, giving only the rotation speed and the intensity in cruise, which must also respect the constraints.

Results : The software gives a solution that satisfies all the constraints : In cruise, the global propulsion efficiency is :

$$\eta_{total} = 69\%$$

To assess the quality of the method, a common method, using the BOUCHERś formula, is applied on the same problem :

- Estimated propeller efficiency : $\eta_{propeller} = 70\%$, then the required power is : $Power_{max} = Power_{useful_{climb}} / \eta_{propeller}$ Then an engine is selected in the data base with a maximum power close to this power
- Knowing that a brushless engine as it best efficiency at about 85% of it maximum speed : $\omega = V \times K_v \times 0.85$
- Estimated propeller airspeed : 25% greater than the airplane airspeed.
- The pitch is computed with the rotation speed and the propeller airspeed.
- The propeller diameter is computed with the BOUCHERś formula. Then the propeller with the closest caracteristics is chosen in the database.
- a controller is selected in the database with a maximal intensity higher than the motor one.

Then the efficiency equations computed previously are applied : In cruise, the global propulsion respect all the constraints, but the global efficiency is only

$$\eta_{total} = 0.55\%$$

In climb, the global propulsion do not satisfy the intensity constraint, and the usefull power has to be reduced of 71% to do so.

Therefore, this example shows that the method can improve the range of an aircraft of about 25%, and solve the climb issues (increase of 30%).

6 Conclusion

Therefore it appears as conclusion of this article that it is possible to improve significantly the propulsive efficiency of an aircraft, and thus its performance, by a judicious choice of its constituting components. This choice is made thanks to the characteristics of the engine, propeller and controller, and an optimization software using a genetic algorithm. The optimisation of the propellant system constituting elements is a first step in the overall optimization of an aircraft.

References

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