

Sensorless adaptive field oriented control of brushless motor

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Abstract—“Field Oriented Control” (FOC) is the the most efficient way to control a brushless motor. This control generates constantly a magnetic field perpendicular to the rotor, which maximises its efficiency. However, to do so, it requires knowledge of the rotor position in real time. The latter can be measured using additional sensors which can be problematic for many reasons. Alternatively, the so-called “sensorless” control consists in the analysis of the electric motor response but needs to know certain characteristics of the engine, compromising compatibility. In this paper, we propose an adaptive control based on the hypothetical rotor position method, which overcomes these problems. This control has also the advantage of remaining optimal although the motor parameters vary over time due to external conditions (e.g. temperature) or aging. The results obtained are very promising and seem to prove its suitability for implementation in real applications.

I. INTRODUCTION

Thanks to the impulse given by recent developments in the fields of drones and electrical radio controlled models, the use of brushless motors has experienced strong growth in recent years. Their numerous advantages over competitors are their high efficiency, superior torque-speed characteristics, compactness and high torque-to-inertia ratio. This explains why they rapidly take the place of brushed DC motors and induction motors [6]. However, the main drawback for these motors is the need for an accurate knowledge of the rotor position. The operating principle of any electric motor is indeed to generate a variable magnetic field in both the rotor and stator that make an angle of around $\frac{\pi}{2}rad$ to each other in order to generate a torque. The aim of the brushes is to maintain this angle while rotating, selecting the coils sequentially. Their removal imposes thus to transfer this duty to another mean, which is electronics for the brushless motor technology. There have been several control methods developed along the years, as listed in [3] [8]. Nevertheless, their generalisation would require a both easy to configure (e.g. “plug and play” like) and low cost control in their drive system. That is why most motor controllers are based upon trapezoidal sensorless control, which consists of powering two motor phases at a time while measuring the back EMF

on the third one to detect the voltage zero-crossing [3] [8]. If this solution offers excellent compatibility between the components, it is not providing the best performance. Indeed, the angle between the magnetic fields of the stator and the rotor is not maintained constant to $\frac{\pi}{2}rad$. Thus, for a given power consumption, the torque is not optimal, which reduces both performance and responsiveness. Therefore, this is not the most optimal way of control. FOC generates indeed the best results up to now. This technique consists of generating a sinusoidal magnetic field vector normal to the stator one in order to maximise the efficiency, which makes it best suited for any three-phases machines, including Permanent Magnet d.c. brushless Motor [8]. In order to measure the rotor position, an additional sensor (Encoders, Resolvers or Hall Effect sensors) is usually added which, in return, increases size and cost, requires extra wiring, complicates the driver electronics, has a limited operational temperature, range and speed and is subject to failures [6] [2]. Advanced control laws have been recently intensively investigated to take away this sensor and only base the control on current and voltage measurements. According to [8], there are two main types of closed loop sensorless control methods for Permanent Magnet d.c. brushless Motor:

- Intrusive sensorless control: based on the machine saliency, it superimposes a high frequency signal to the basic phase voltages and currents. This method presents drawbacks: first, it requires a good saliency ratio and then, the inverter switches age prematurely due to their intensive use.
- Sensorless control based on Back-EMF measurement: this method suffers of bad reliability at standstill or very low-speed. However it seems to be the cheapest way of control and looks ideal for all applications with relatively high operating minimum speed.

Various sensorless control techniques have been developed based on Back-EMF measurement. However, they are all based on the knowledge of the motor parameters. The control must therefore be tweaked for every system it deals with, which is unacceptable for any large scale use. Some adaptive controls, such as the one presented in [5], have been even proposed, but they still need the motor parameters as the adaptive techniques are only used to estimate the rotor position. The present article proposes a direct adaptive control without relying on motor characteristics. It also presents the advantage of estimating the parameters of the motor, which enables their further exploitation. In addition, it compensates the parameters evolution due, for instance, to the ageing or the temperature elevation while running.

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The article shows how the control is obtained, starting from the initial motor electrical model. In Section II, the electrical model of the BDSM motor is exposed. In Section III, an adaptive direct control is proposed. In Section IV, the results of the simulation of the proposed control are exposed. Section V concludes this article.

II. BDSM MODELISATION

A. Electrical modelisation

The study of the brushless motor starts with the following electrical model [9] [10]. This electrical model is based on the electrical diagram, shown in Fig. 1¹.

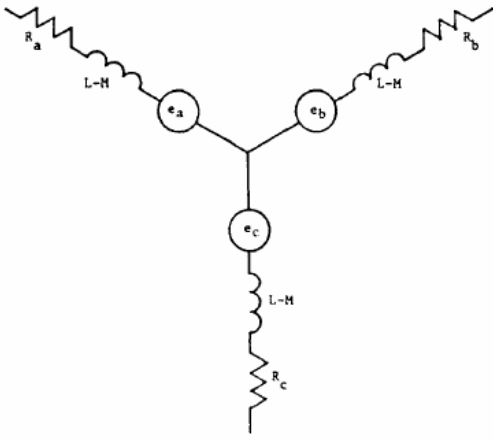


Fig. 1. Equivalent circuit from electric equation, (courtesy of Pillay [9])

$$\begin{pmatrix} u_a \\ u_b \\ u_c \end{pmatrix} = \begin{pmatrix} R & 0 & 0 \\ 0 & R & 0 \\ 0 & 0 & R \end{pmatrix} \begin{pmatrix} i_a \\ i_b \\ i_c \end{pmatrix} + \begin{pmatrix} L & 0 & 0 \\ 0 & L & 0 \\ 0 & 0 & L \end{pmatrix} \frac{d}{dt} \begin{pmatrix} i_a \\ i_b \\ i_c \end{pmatrix} - p \cdot \Omega \cdot \phi_r \begin{pmatrix} \sin(p \cdot \theta) \\ \sin(p \cdot \theta - \frac{2\pi}{3}) \\ \sin(p \cdot \theta + \frac{2\pi}{3}) \end{pmatrix} \quad (1)$$

where a, b and c are the three motor phases, u, i, R and L are respectively the phase voltage, current, resistor and inductance (respectively in V, A, Ω and H), p is the number of poles, Ω is the rotation velocity (in $rad \cdot s^{-1}$), ϕ_r is the rotor magnetic flux (in *Weber*) and θ is the rotor position (in *rad*).

B. $\alpha \beta \gamma$ transformation

In order to reduce the control computing time, the $\alpha \beta \gamma$ (or Clarke) transformation can be applied. This transformation, used for most three-phase circuits, enables the control of only two equivalent phases instead of three. It consists of passing from the initial a, b, c referential to the $\alpha \beta$ reference frame applying the following transformation

¹As explained in [9], Fig. 1 can be simplified by substituting $L - M$ by L and assuming $R_a = R_b = R_c$.

$$\text{matrices: } P_{\alpha\beta}^{abc} = \frac{2}{3} \begin{bmatrix} 1 & -\frac{1}{2} & -\frac{1}{2} \\ 0 & \frac{\sqrt{3}}{2} & -\frac{\sqrt{3}}{2} \end{bmatrix}, P_{abc}^{\alpha\beta} = P_{\alpha\beta}^{abc^{-1}} = \begin{bmatrix} 1 & 0 \\ -\frac{1}{2} & \frac{\sqrt{3}}{2} \\ -\frac{1}{2} & -\frac{\sqrt{3}}{2} \end{bmatrix}$$

This point is critical since FOC has to work at very high frequency and the computation relative to the adaptation is quite heavy compared to a basic control.

Equation (1) becomes:

$$\begin{pmatrix} u_\alpha \\ u_\beta \end{pmatrix} = \begin{pmatrix} R & 0 \\ 0 & R \end{pmatrix} \begin{pmatrix} i_\alpha \\ i_\beta \end{pmatrix} + \begin{pmatrix} L & 0 \\ 0 & L \end{pmatrix} \frac{d}{dt} \begin{pmatrix} i_\alpha \\ i_\beta \end{pmatrix} + p \cdot \Omega \cdot \phi_r \begin{pmatrix} -\sin(p \cdot \theta) \\ \cos(p \cdot \theta) \end{pmatrix} \quad (2)$$

$$\text{where } \begin{pmatrix} u_\alpha \\ u_\beta \end{pmatrix} = P_{\alpha\beta}^{abc} \begin{pmatrix} u_a \\ u_b \\ u_c \end{pmatrix} \text{ and } \begin{pmatrix} i_\alpha \\ i_\beta \end{pmatrix} = P_{\alpha\beta}^{abc} \begin{pmatrix} i_a \\ i_b \\ i_c \end{pmatrix}$$

III. CONTROL CONSTRUCTION

The present work aims at designing an adaptive control law that, on the one hand, estimates the two almost constant parameters R and L , to adapt itself perfectly to the motor it manages, and on the other hand, estimates the third term of the addition of equation (2) in order to extract the rotor position θ required by any brushless control as stated in Section I. Designing the control in the present reference frame seems to be the best choice. It is indeed the most reduced one, voltages and currents being likely to be sinusoidal at high frequency, which would approach the persistent excited condition. In addition, on a parameter exploitation point of view, the inverse Clark transformation would help estimate the different phase real parameters and thus detects more precisely any degradation.

When the motor is working optimally, the dynamic of the current is synchronised to the one of the rotor magnetic field. The second member of equation (2) can be therefore written as:

$$p \cdot \Omega \cdot \phi_r \begin{pmatrix} -\sin(p \cdot \theta) \\ \cos(p \cdot \theta) \end{pmatrix} = \begin{pmatrix} k_1 & k_2 \\ k_3 & k_4 \end{pmatrix} \begin{pmatrix} i_\alpha \\ i_\beta \end{pmatrix}$$

where k_1, k_2, k_3 and k_4 are constants.

Thus, the electrical equation (2) can be rewritten as:

$$\begin{pmatrix} u_\alpha \\ u_\beta \end{pmatrix} = \begin{pmatrix} k_1 + R & k_2 \\ k_3 & k_4 + R \end{pmatrix} \begin{pmatrix} i_\alpha \\ i_\beta \end{pmatrix} + \begin{pmatrix} L & 0 \\ 0 & L \end{pmatrix} \frac{d}{dt} \begin{pmatrix} i_\alpha \\ i_\beta \end{pmatrix} \quad (3)$$

As shown in equation (3), the considered reference frame is not suitable for an adaptive control implementation. The adaptive part of this control aims indeed at reducing the error of the control varying the different parameters. Therefore, it will tend to overestimate the value of the resistor matrix

which is constant, in order to set to zero the magnetic flux term that is sinusoidal and thus increases the error. This is prohibitive not only because of the loss of the motor parameters estimation, but also because the stator magnetic field term is required to estimate the rotor position. It may be noticed that, the problem would be the same in the initial a, b, c referential.

The referencial frame must be thereby substituted, one more time, by a new one without any correlation between the different components. In order to do so, a variant of the $d q 0$ transformation is used.

A. Modified $d q 0$ transformation

A variant of the $d q 0$ transformation (or Park) is applied considering the estimated motor rotor position $\hat{\theta}$. This transformation removes the sinusoidal nature of the current, voltage and magnetic field terms [3]. The transformation matrix between the two last referential are [11] [7] [4].

$$P_{dq}^{\alpha\beta} = \begin{pmatrix} \cos(p \cdot \hat{\theta}) & \sin(p \cdot \hat{\theta}) \\ -\sin(p \cdot \hat{\theta}) & \cos(p \cdot \hat{\theta}) \end{pmatrix} \text{ and } P_{\alpha\beta}^{dq} = \begin{pmatrix} \cos(p \cdot \hat{\theta}) & -\sin(p \cdot \hat{\theta}) \\ \sin(p \cdot \hat{\theta}) & \cos(p \cdot \hat{\theta}) \end{pmatrix}$$

Defining $\tilde{\theta}$ as the error of the rotor position estimation, it comes: $\tilde{\theta} = \hat{\theta} - \theta$

Following the same method as exposed in [7], the electric model (2) becomes:

$$\begin{pmatrix} u_d \\ u_q \end{pmatrix} = \begin{pmatrix} L_d & 0 \\ 0 & L_q \end{pmatrix} \begin{pmatrix} \dot{i}_d \\ \dot{i}_q \end{pmatrix} + \begin{pmatrix} R_d & -L_q \cdot p \cdot \hat{\Omega} \\ L_d \cdot p \cdot \hat{\Omega} & R_q \end{pmatrix} \begin{pmatrix} i_d \\ i_q \end{pmatrix} + p \cdot \Omega \cdot \phi_r \cdot \begin{pmatrix} \sin(p \cdot \hat{\theta}) \\ \cos(p \cdot \hat{\theta}) \end{pmatrix}$$

It should be noted that L_d and L_q are now segregated for generalisation reasons since they may vary a bit depending on the saliency of the motor. Nevertheless, classical aero-model brushless motors still verify the property: $L = L_d = L_q$.

This latter equation can be written as:

$$U = A\dot{I} + BI + E \quad (4)$$

Where: $U = \begin{pmatrix} u_d \\ u_q \end{pmatrix}$, $I = \begin{pmatrix} i_d \\ i_q \end{pmatrix}$, $A = \begin{pmatrix} L_d & 0 \\ 0 & L_q \end{pmatrix}$, $B = \begin{pmatrix} R_d & -L_q \cdot p \cdot \hat{\Omega} \\ L_d \cdot p \cdot \hat{\Omega} & R_q \end{pmatrix}$ and

$$E = \begin{pmatrix} E_d \\ E_q \end{pmatrix} = p \cdot \Omega \cdot \phi_r \cdot \begin{pmatrix} \sin(p \cdot \hat{\theta}) \\ \cos(p \cdot \hat{\theta}) \end{pmatrix} \quad (5)$$

This is the final electric model used in the rest of the article.

B. Adaptive control design

An adaptive control law based on the direct adaptive control method [1] is now proposed .

Control idea: The main idea of the present control is to consider that the mechanical dynamic of the motor is much slower than the electrical one. Therefore on an electrical time scale, E , which depends on the motor rotational speed Ω and the rotor drift $\tilde{\theta}$ can be considered as constant and be estimated as an unknown parameter by the adaptative control.

The design starts with the definition of the control law in Section III-B.1. Then the error of the control is estimated in Section III-B.2. A Lyapunov law is proposed in Section III-B.3 and the adaptive law is extracted in order to estimate the matrices A, B and E of equation (4). The rotor position is lastly estimated from parameter E in Section III-B.4.

1) *Control law definition:* The aim is to follow a desired current trajectory T . The error of the control Δ is defined as follows:

$$\Delta = T - I \quad (6)$$

It is possible to superimpose a white noise to the trajectory T in order to help the convergence of the parameters.

The aim of the control is to reduce the magnitude of Δ . In order to do so, the following relation is proposed to be satisfied by the error control:

$$\dot{\Delta} = -K\Delta \quad (7)$$

where K is the control gain defined positive.

From equations (6) and (7), it comes:

$$\begin{aligned} \dot{T} - \dot{I} &= -K\Delta \\ \Rightarrow A\dot{I} &= A(K\Delta + \dot{T}) \end{aligned} \quad (8)$$

Then substituting equation (8) in equation (4), it comes:

$$U = A(K\Delta + \dot{T}) + BI + E \quad (9)$$

As the actual control depends on the estimated parameters, noted \hat{A}, \hat{B} and \hat{E} , rather than the real values, equation (9) becomes :

$$U = \hat{A}(K\Delta + \dot{T}) + \hat{B}I + \hat{E} \quad (10)$$

2) *Control error estimation:* Now including the estimation errors of the different parameters: $\tilde{A} = \hat{A} - A$, $\tilde{B} = \hat{B} - B$, $\tilde{E} = \hat{E} - E$, equation (10) becomes:

$$U = \tilde{A}(K\Delta + \dot{T}) + \tilde{B}I + \tilde{E} + A(K\Delta + \dot{T}) + BI + E \quad (11)$$

Then inserting the equation (4), in the derivative of the estimation error expression (6) $\dot{\Delta} = \dot{T} - \dot{I}$, it comes:

$$A\dot{\Delta} = A\dot{T} + BI + E - U \quad (12)$$

Then substituting equation (11) in equation (12) ²:

$$\begin{aligned} A\dot{\Delta} &= A\dot{T} + BI + E - \tilde{A}(K\Delta + \dot{T}) \\ &\quad - \tilde{B}I - \tilde{E} - AK\Delta - A\dot{T} - BI - E \\ \Leftrightarrow \dot{\Delta} &= -K\Delta - A^{-1}(\tilde{A}(K\Delta + \dot{T}) + \tilde{B}I + \tilde{E}) \end{aligned} \quad (13)$$

²Matrix A being diagonal and strictly positive, it is inversible.

Defining: $\tilde{\lambda}^T = (\tilde{A} \quad \tilde{B} \quad \tilde{E})$, and: $\eta = \begin{pmatrix} K\Delta + \dot{T} \\ I \\ 1 \end{pmatrix}$, equation (13) becomes:

$$\dot{\Delta} = -K\Delta - A^{-1}\tilde{\lambda}^T\eta$$

3) *Adaptive law based on Lyapunov function:* The following Lyapunov function candidate is proposed to define the stability condition of the control [12]:

$$V = \frac{1}{2}\Delta^T A \Delta + \frac{1}{2}tr(\tilde{\lambda}^T \Gamma^{-1} \tilde{\lambda}) \quad (14)$$

Where Γ is a real positive definite diagonal matrix. Deriving equation (14), it comes:

$$\begin{aligned} \dot{V} &= \Delta^T A \dot{\Delta} + tr(\tilde{\lambda}^T \Gamma^{-1} \dot{\tilde{\lambda}}) \\ &= -\Delta^T A K \Delta - \Delta^T A A^{-1} \tilde{\lambda}^T \eta + tr(\tilde{\lambda}^T \Gamma^{-1} \dot{\tilde{\lambda}}) \\ &= -\Delta^T A K \Delta - tr(\tilde{\lambda}^T \eta \Delta^T) + tr(\tilde{\lambda}^T \Gamma^{-1} \dot{\tilde{\lambda}}) \\ &= -\Delta^T A K \Delta - tr(\tilde{\lambda}^T (\eta \Delta^T - \Gamma^{-1} \dot{\tilde{\lambda}})) \end{aligned}$$

In order to have: $\dot{V} < 0$, the following relation can be imposed:

$$\begin{aligned} \eta \Delta^T - \Gamma^{-1} \dot{\tilde{\lambda}} &= 0 \\ \Leftrightarrow \dot{\tilde{\lambda}} &= \Gamma \eta \Delta^T \end{aligned} \quad (15)$$

Which represents the adaptive part of the control.

4) *Rotor position estimation:* The estimation of the rotor position is determined from equation (5):

- if $\hat{E}_d \geq \hat{E}_q$:

$$\begin{aligned} \frac{\hat{E}_q}{\hat{E}_d} &= \frac{\Omega \cdot \phi_r \cdot \cos(p \cdot \tilde{\theta})}{\Omega \cdot \phi_r \cdot \sin(p \cdot \tilde{\theta})} \\ \Leftrightarrow p \cdot \tilde{\theta} &= \cot^{-1}\left(\frac{\hat{E}_d}{\hat{E}_q}\right) \end{aligned}$$

- if $\hat{E}_d < \hat{E}_q$

$$\begin{aligned} \frac{\hat{E}_d}{\hat{E}_q} &= \frac{\Omega \cdot \phi_r \cdot \sin(p \cdot \tilde{\theta})}{\Omega \cdot \phi_r \cdot \cos(p \cdot \tilde{\theta})} \\ \Leftrightarrow p \cdot \tilde{\theta} &= \tan^{-1}\left(\frac{\hat{E}_d}{\hat{E}_q}\right) \end{aligned}$$

Following the method proposed in [2], a control like PI is applied to estimate the rotation speed evolution:

$$p \cdot \hat{\Omega} = -K_p \cdot p \cdot \tilde{\theta} - K_i \cdot \int_{t_0}^t p \cdot \tilde{\theta} \quad (16)$$

Lastly, the stator position $\hat{\theta}$ is obtained integrating $\hat{\Omega}$:

$$p \cdot \hat{\theta} = \int_{t_0}^t p \cdot \hat{\Omega} \quad (17)$$

It is thus possible, using the set of equations (10), (15), (16) and (17) to estimate the required position and speed of the rotor. The only condition is to have a sufficient rotating speed in order to be able to measure E . To reach this minimum rotation speed from stop, a classical open loop sensorless control can be used [8], however this is beyond the scope of this article.

IV. SIMULATION RESULTS

The motor with the following characteristics is simulated using Scicos³ software: $P = 5$, $R_d = 110 \cdot 10^{-3} \Omega$, $R_q = 90 \cdot 10^{-3} \Omega$, $L_d = 55 \cdot 10^{-6} H$, $L_q = 60 \cdot 10^{-6} H$ and $\phi_r = 0.00012 Wb$

These values are typical for a small-size RC-model brush-less motor.

This motor is controlled to obtain the desired path T , as shown in Fig. 2:

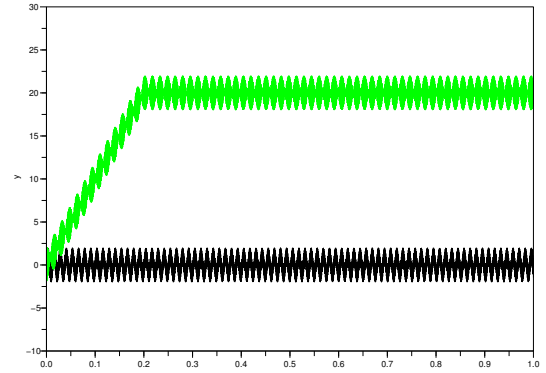


Fig. 2. Desired path T_d in black and T_q in green, A vs t

One can notice that a noise has been superimposed all over the initial intended path. This is done to accelerate the convergence of the different parameters. This noise has been set at a fifth of the expected path on the simulation, but the amplitude must be tweaked depending on the required convergence velocity: the higher is the noise, the faster is the convergence.

It has to be kept in mind that this noise is only necessary when the motor is started and can be deleted when the parameters have converged. For instance, a white noise could be imposed alone at the very beginning, before starting the real task, but the engine operation strategy is beyond the scope of the present article.

The obtained path and the error are presented on Fig. 3 and Fig. 4:

It can be noticed that the convergence is faster than a second. The gain K of the path following control part has been set to: $K = 10^4 \cdot \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$

The evolution of the parameter A estimation is shown on Fig. 5. To obtain such an evolution, the gain Γ_A of the adaptive control part has been set to: $\Gamma_A = 10^{-5}$. This value is much smaller than the following one. It is done so because

³<http://www.scicos.org/>

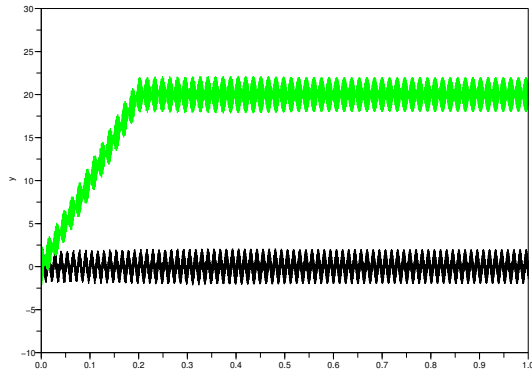


Fig. 3. Obtained path, I_d in black and I_q in green, A vs s

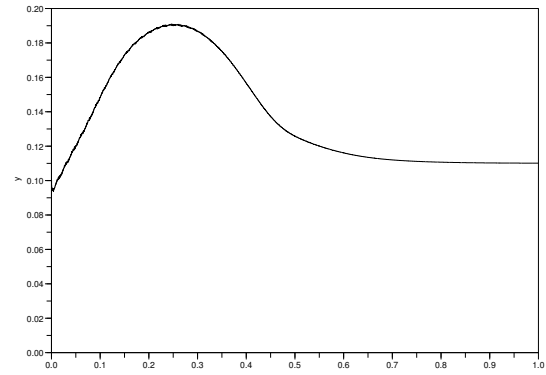


Fig. 6. Parameter R_d estimation, ω vs s

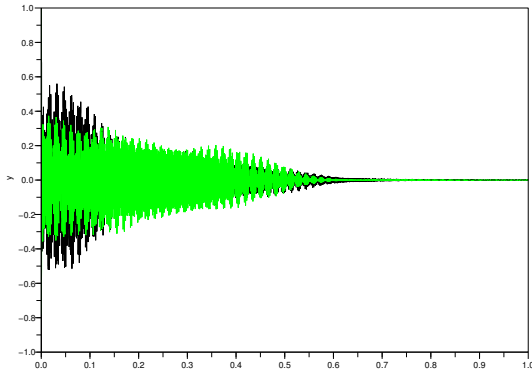


Fig. 4. Error evolution, Δ_d in black and Δ_q in green, A vs s

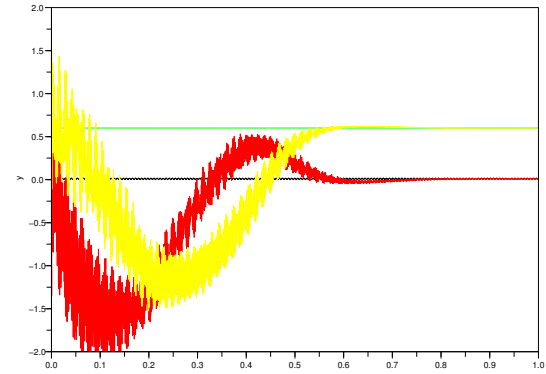


Fig. 7. Parameter E estimation, E_d real and estimated respectively in black and red, E_q real and estimated respectively in green and yellow, ω vs s

of the very small size of a parameter compared to the two others.

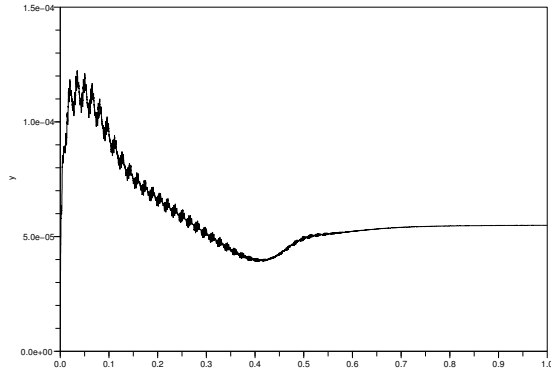


Fig. 5. Parameter L_d estimation, H vs s

In the same manner, the evolution of the parameter B estimation is shown on Fig. 6. The gain Γ_B is here set to : $\Gamma_B = 10$.

Lastly, the evolution of the estimation of the most important parameter, E, is shown in Fig. 7. Zoom on the converged part is visible in the Fig. 8. The gain Γ_E is here set to : $\Gamma_E = 10^4$. It is set much higher than the two others in order to compensate the fact that E is not any more constant.

It can be noticed on Fig. 8 that a drift has been superimposed to the time linear position of the motor in order to simulate the brutal fluctuation of the torque applied to the

motor. However, the convergence of the motor position estimation is both fast and precise, which makes it suitable for performing the necessary rotor position estimation required by the field oriented control.

V. CONCLUSION

It has been seen along this article that it is possible to control a brushless motor without requiring any knowledge about the parameters of the controlled motor. To do so, it is necessary to work in a rotating reference frame defined by the estimated position of the rotor in order to avoid synchronisation between the different dynamics which would lead to the loss of the rotor position. The control tends to converge very rapidly providing quickly the actual parameter values which can then be post-processed. Moreover, the rotor position is always well estimated and therefore never affects the quality of the motor operation. Nevertheless, it appears that the control estimates several times the same parameters (L_d , L_q ...) as well as some known parameters (the two zeros of matrix A), which is consuming a lot of unnecessary computation resources. Further work will consist in optimising the adaptive part of the control in order to both reduce the computational time and make the estimation more robust.

Lastly, another significant advantage the control has, but which will not be detailed in this article, is the precise tracking of the motor condition, which could be highly

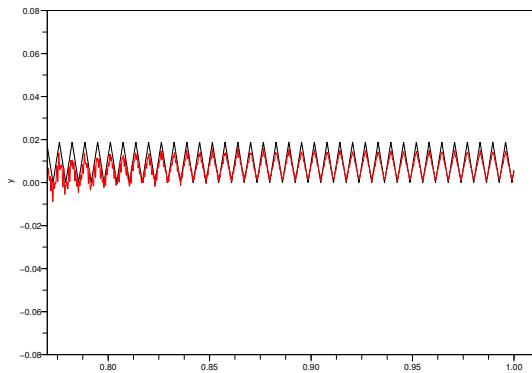


Fig. 8. Parameter E estimation

valuable when considering its maintenance, as introduced in [3].

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